Gravitational Collapse and Neutrino Emission of Massive Stars

Ken’ichiro Nakazato

A doctoral dissertation submitted to
Department of Physics,
Waseda University

July 2008
Abstract

Stars with mass of larger than 10 solar masses become gravitational collapse at the end of their lives. Some of them make explosion and produce core collapse supernovae. Very massive stars, which can not make explosion, are thought to form black hole by the collapse. In fact, there are many black hole candidates with the masses from several to $10^9$ solar masses in the universe whereas their origins are unclear. On the other hand, detections of neutrinos from supernova 1987A have declared that the neutrino astronomy is practical to study high-energy astrophysical phenomena. Neutrinos can bring us useful information not only on astrophysics but also on other fundamental physics, such as particle physics, hadron physics and nuclear physics. In particular, the collapse involving the black hole formation is thought to undergo very hot and dense state in the core comparing with ordinary supernovae.

In this thesis, we perform the numerical simulations of gravitational collapse of massive stars and black hole formation. We solve numerically the general relativistic hydrodynamics and neutrino transfer equations simultaneously, treating neutrino reactions in detail under spherical symmetry. Owing to the detailed computation, we can evaluate the neutrino flux precisely. From the results of our simulations, we discuss (1) how to probe the black hole progenitors, and (2) whether we can investigate the physics of hot dense matter. In both cases, we suppose to utilize “neutrino signal” for investigation. For the former question, we find that the neutronization burst of neutrinos is less remarkable or disappears completely for more massive models. This is because the density is lower at the bounce. We also find that the spectra of neutrinos, except the electron type, are softer owing to the electron-positron pair creation before the bounce. Moreover, we estimate the flux of relic neutrino from the first stars in the universe, which is thought to be very massive as 300-10,000 solar masses. As a result, while the detection of these neutrinos is difficult for the currently operating detectors, we find that the spectrum will enable us to obtain the information on the formation history of the first stars. As for the latter question, we examine the effects of quarks and pions. As a result, we find that the duration time of the neutrino emission becomes shorter for the model with pions and quarks. Then the total energy of the emitted neutrinos becomes lower. The neutrino luminosity rises under the effect of pions since the density of the proto-neutron star becomes high. In this thesis we show that the neutrino astronomy is useful not only for the supernovae but also for the black hole formation.
Acknowledgements

The author wishes to express his gratitude to his supervisor, Prof. S. Yamada for giving him an opportunity to study on this theme, stimulating discussions and continuous encouragement. He would also like to express his appreciation to his collaborator, Prof. K. Sumiyoshi for many helpful suggestions and aids in computations, which are indispensable to this work. He is grateful to Profs. H. Suzuki and A. Ohnishi for fruitful discussions about the subjects of this thesis. He is owing Prof. H. Umeda for useful information on supernova progenitors and some numerical data. He wishes to express his thank to Dr. S. Ando for advice on the supernova relic neutrino background, Dr. C. Ishizuka for advice on the equation of state, Prof. Y. Koma for advice on the QCD transition, Dr. H. Maeda for advice on the general relativity, Prof. T. Mitsui for advice on the neutrino detector and Dr. T. Ohkubo for advice on the Population III massive stars. He also thanks deeply Profs. S. A. Colgate, T. Daishido, C. L. Fryer, K. Fukushima, T. Hanawa, K. Iida, H.-Th. Janka, M. Liebendörfer, K. Maeda, E. Müller, K. Oyamatsu and M. Takano for their interests and/or comments on his study. He is thankful to be inspired by Drs. H. Ito, H. Sawai and all the other members of the High Energy AstroPhysics group in Waseda University, including his former colleagues.

In this study, numerical computations were partially performed on the VPP5000 at the Center for Computational Astrophysics (CfCA) of the National Astronomical Observatory of Japan (NAOJ), the Altix3700 at Yukawa Institute for Theoretical Physics (YITP) and the SX-8R at Research Center for Nuclear Physics (RCNP) of Osaka University. The author acknowledges the support by Japan Society for Promotion of Science (JSPS) Research Fellowship and the 21st-Century COE Program “Holistic Research and Education Center for Physics of Self-organization Systems” at Waseda University.
# Contents

1 Introduction ............................................. 1
   1.1 Overview ........................................... 1
   1.2 Stellar Evolution ...................................... 1
       1.2.1 Population I stars ............................... 2
       1.2.2 Population III stars ............................ 3
   1.3 Stellar Collapse and Supernova Explosion ............... 4
   1.4 Black Hole Formation .................................. 7
   1.5 Intermediate Mass Black Holes ........................ 9
   1.6 Neutrino Astronomy .................................... 10
   1.7 Quark Matter in Astrophysics .......................... 12
   1.8 Aims of this Thesis .................................... 13

2 Neutrino Radiation Hydrodynamics .......................... 15
   2.1 Basic Equations ...................................... 15
   2.2 Criterion for Apparent Horizon ......................... 18
   2.3 Treatment of Neutrinos ................................ 18

3 Equation of State ......................................... 21
   3.1 Relativistic EOS of Nuclear Matter ..................... 21
   3.2 Setups for Hadron-Quark Mixed EOS ..................... 22
       3.2.1 EOS for Pure Hadronic Matter .................. 22
       3.2.2 EOS for Pure Quark Matter ...................... 24
       3.2.3 EOS for Hadron-Quark Mixed Phase .............. 26
       3.2.4 Treatment of Muons ............................... 27
   3.3 Results of Hadron-Quark Mixed EOS ..................... 29
       3.3.1 Assessments of EOS ............................... 29
       3.3.2 Maximum Mass of the Hybrid Stars .............. 35

4 Systematic Study of Stellar Collapse ........................ 39
   4.1 Initial Models ....................................... 39
       4.1.1 Setups ........................................... 39
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1.2</td>
<td>Natural Counterparts of the Models</td>
<td>40</td>
</tr>
<tr>
<td>4.2</td>
<td>Results</td>
<td>42</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Dynamical Features</td>
<td>43</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Neutrino Signals</td>
<td>47</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Initial Velocity Dependence</td>
<td>54</td>
</tr>
<tr>
<td>4.3</td>
<td>Astrophysical Implications</td>
<td>55</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Detection of Low Energy $\bar{\nu}_e$ by SK III and KamLAND</td>
<td>55</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Detection of Neutronization Burst by SNO</td>
<td>56</td>
</tr>
<tr>
<td>5</td>
<td>Collapse of Population III Stars</td>
<td>59</td>
</tr>
<tr>
<td>5.1</td>
<td>Dynamical Features and Neutrino Signals</td>
<td>59</td>
</tr>
<tr>
<td>5.2</td>
<td>Relic Neutrinos from Population III Stars</td>
<td>71</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Formulation for Relic Neutrino Flux</td>
<td>72</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Results and Discussion</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>Collapse Including Quarks and Pions</td>
<td>79</td>
</tr>
<tr>
<td>6.1</td>
<td>Initial Models</td>
<td>79</td>
</tr>
<tr>
<td>6.2</td>
<td>Results and Discussions</td>
<td>82</td>
</tr>
<tr>
<td>7</td>
<td>Conclusions</td>
<td>89</td>
</tr>
<tr>
<td>A</td>
<td>Convergence Check</td>
<td>93</td>
</tr>
<tr>
<td>B</td>
<td>Neutrino Oscillation</td>
<td>95</td>
</tr>
<tr>
<td>B.1</td>
<td>General Formulation of Flavor Conversion by MSW Effect</td>
<td>96</td>
</tr>
<tr>
<td>B.2</td>
<td>Flavor Conversion inside Stellar Envelope</td>
<td>97</td>
</tr>
<tr>
<td>B.3</td>
<td>Progenitor Dependence</td>
<td>100</td>
</tr>
<tr>
<td>B.4</td>
<td>Earth Effects: Nadir Angular Dependence</td>
<td>102</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Overview

There are various types of stars in the universe. Some stars are very massive, and others are little. Some stars are single, and others are binaries. Stars formed in the early universe may different from stars of later generations. These stars are usually steady but sometimes make explosive events. In particular, stars whose masses are several times larger than the solar mass ($M_\odot$) are known to make a gravitational collapse at the end of their lives. As a result, the collapse is either halted to produce supernova explosion or followed to the black hole formation. Supernova explosion is not merely the spectacular phenomenon but also one of the crucial events for the evolution of the galaxy. They scatter the metal, which is heavier element than helium, and these elements become constituents of stars of next generations, planets and lives. The mechanism of core-collapse supernova has been intensively studied so far (e.g., Colgate & White 1966) although the definitive results are still absent. Black holes are also important members in the universe. Many galaxies including our galaxy have a supermassive black hole (SMBH) at their center and some of them may be a central engine of the intergalactic jets. In this thesis, we mainly focus on the other branch of stellar collapse, the black hole formation. In the following, we introduce some topics on astrophysics related with this thesis and we state the aims of this thesis at last.

1.2 Stellar Evolution

When the star begins to collapse, its mass is not same as the initial one. This is because it loses its mass during the evolution. The mass loss rate
depends mainly on the stellar metalicity, which is a mass fraction of metal. Thus the fate of a star is decided by the initial mass and metalicity. Whereas stellar rotation and binary interactions may be also important for the stellar evolution, their effects are not well studied. Because metal is not synthesized at the big bang and made only in the stars, the first stars formed in the universe are metal free and they are called Population III (Pop III) stars. The metalicity of the sun, which is one of the stars of later generations, is $Z \sim 0.02$ and these stars are called Population I (Pop I) stars. Not only the evolutions but also the initial mass functions (IMF) of Pop I and Pop III stars are different owing to the difference of metalicity. In this section, we briefly review the standard understandings on them.

1.2.1 Population I stars

Stars are formed when the dense region of the circum stellar cloud starts contraction. As the matter is contracted inside the star, its temperature rises. When the central temperature, $T_c$, becomes over $10^7$ K, hydrogen atoms are ignited to synthesize helium atoms and the contraction is halted. Stars in this stage are called main sequence star. If the stellar mass is larger than $10 M_\odot$, the central temperature continues to rise and starts helium, carbon, oxygen and silicon burning successively. As a result, the iron core is formed at the center. Inside the iron core, all nuclear reactions become equilibrium to inverse reactions of themselves and various elements called “iron”, which are Fe, Ni, Co, Mn and so on, are created. This stage is called Nuclear Statistical Equilibrium (NSE). When the central temperature becomes $T_c \gtrsim 5 \times 10^9$ K, photodisintegration reaction of nucleus occurs. This reaction makes the iron core gravitationally instable and the iron core begins to collapse. On the other hand, stars less than $10 M_\odot$ can not raise the temperature sufficiently. They halt the burning and become white dwarfs losing their envelope. However, stars with 8-10$ M_\odot$ may become gravitationally unstable by the electron capture reaction to make ONeMg supernovae explosion, while their fate is still uncertain. The evolutionary tracks of Pop I stars are shown in Figure 1.1. It is noted that we can not neglect the effects of mass loss for Pop I stars. The mass loss by radiative wind is thought to be efficient for massive metal-rich stars. While, unfortunately, the metallicity dependence of the mass-loss rate is unclear well, one estimation has suggested that a star whose initial mass and metallicity are $35 M_\odot$ and $Z_\odot$, respectively, loses its hydrogen envelope and ends in Wolf-Rayet star with 5-8$ M_\odot$ (Woosley et al. 2002).
1.2.2 Population III stars

Studies of Pop III stars are indispensable to understand the star formation history of the universe and the galactic chemical evolution. Pop III stars are more difficult to study than Pop I stars because they are the first stars formed in the universe and there are no direct observations of them. However, recently, lots of observational information on them have been obtained from WMAP (e.g., Spergel et al. 2003), ground-based and airborne observations of very metal-poor stars (e.g., Frebel et al. 2005), and so on. On the other hand, theoretical studies of the first star formation suggest that Pop III stars may have a large population of very massive objects from hundreds to thousands solar mass (e.g., Nakamura & Umemura 2001) although their IMF and star formation rate are still uncertain. The evolution and fate of Pop III massive stars have also been studied for a long time (e.g., Bond et al. 1984; Fryer et al. 2001). According to these studies, Pop III massive stars lose little of their mass during the quasi-static evolutions because of zero-metallicity.

If an initial mass of the Pop III star is larger than $\sim 100M_\odot$, its evolution and fate are very different from Pop I stars. After the central helium depletion, electron-positron pairs are created copiously at the center of the star.
CHAPTER 1. INTRODUCTION

This process consumes a part of thermal energy for the electron-positron rest mass, making the star unstable against gravitational collapse. It is called the pair-instability. The stellar collapse induced by the pair-instability increases the density and temperature and ignites oxygen. If the stellar mass is less than $\sim 260 M_\odot$, the rapid nuclear burning reverses the collapse and disrupts a whole star, giving rise to the so-called pair-instability supernova. On the other hand, more massive stars can not halt the collapse by nuclear burnings and emit a large amount of neutrinos before forming a black hole (e.g., Heger et al. 2003). Note that, however, this threshold is still uncertain at present (e.g., Ohkubo et al. 2006). These black holes will eventually have the mass of their progenitor. It is noted that the mass range is consistent with that of intermediate mass black holes, whose existence is recently suggested by observations. While their origin is unknown at present, some authors indicate that they are remnants of Pop III massive stars (e.g., Mii & Totani 2005). One of the other candidates for them is stated in §1.5.

1.3 Stellar Collapse and Supernova Explosion

When a star can not support its self gravity with pressure, gravitational collapse is caused. In order to judge whether the star is gravitationally stable or not, the structure adiabatic index, $\bar{\gamma}$, is useful. $\bar{\gamma}$ is calculated by the adiabatic index, $\gamma$, which is defined as,

$$\gamma \equiv \left( \frac{\partial \ln p}{\partial \ln \rho} \right)_s,$$

(1.1)

where $p$, $\rho$ and $s$ are the pressure, density and entropy, respectively. The structure adiabatic index for spherically symmetric star is calculated as,

$$\bar{\gamma} = \frac{\int_0^R 4\pi r^2 p \gamma \, dr}{\int_0^R 4\pi r^2 p \, dr},$$

(1.2)

where, $r$ is a radial coordinate and $R$ is a radius of the star. When the inequality, $\bar{\gamma} < 4/3$, is satisfied, the star in a equilibrium configuration is unstable for the radial adiabatic perturbation. Similarly, we can see that the location where $\gamma < 4/3$ is locally unstable inside the star. The gravitationally unstable regions in the density and temperature plane ($\rho - T$ plane) are shown in Figure 1.2. Pop I stars with $\gtrsim 10 M_\odot$ start to collapse from the “photodisintegration” region. The red line in Figure 1.2 is the evolution of the central density and temperature of the supernova whose progenitor is $15 M_\odot$. On the other hand, Pop III stars with $\gtrsim 100 M_\odot$ start to collapse from the “pair” region.
In the following, we summarize the standard scenario of the core-collapse supernovae for Pop I stars. At the onset of collapse, the electron capture on nuclei occurs inside the core and electron type neutrinos ($\nu_e$) are emitted. As the core density increases, the neutrino mean free path becomes shorter and the neutrino sphere is formed. In order to define the neutrino sphere, the neutrino optical depth in the star is needed. At the radius $r$, the neutrino optical depth, $d(r)$, is expressed as,

$$d(r) = \int_r^{R_s} \frac{dr'}{l_{\text{mfp}}(r')}$$

where $R_s$ is the stellar radius and $l_{\text{mfp}}$ is the mean free path of neutrinos. The radius of neutrino sphere, $R_\nu$, is defined as the radius which satisfies,

$$d(R_\nu) = \frac{2}{3}$$
Figure 1.3: Summary of supernova mechanism and neutrino emission from Suzuki (1990).
Because the mean free path depends on the energy and species of neutrinos, so does the neutrino sphere. Here, we mention the neutrino sphere of the neutrinos with the mean energy. The collapse is bounced when central density exceed the nuclear density and the shock wave is formed. This shock wave propagates outside dissociating nuclei into free nucleons. Because the cross section of the electron capture on free proton is lager than that of on nuclei, huge number of $\nu_e$ are generated. When the shock wave go through the neutrino sphere, these $\nu_e$ are emitted. This emission is called neutronization burst. The shock wave may be once stalled and revived by the neutrino heating. Finally the shock wave reach at the surface of the star and supernova explosion occurs. On the other hand, matter behind the shock wave accretes to the inner core. It is called a protoneutron star and becomes the neutron star via the neutrino cooling. These features are illustrated in Figure 1.3.

Observationally, supernovae are characterized by the main sequence mass of the progenitor and the ejected $^{56}$Ni mass. In Figure 1.4, several supernovae observed recently are plotted. From this figure, we can see that supernovae split in two branches, namely a hypernova branch and a faint-supernova branch, for the mass range $\gtrsim 25M_\odot$. This trend may indicate the threshold of the black hole formation is somewhere around this mass and the rotation makes the difference of two branches. In other words, the strongly rotating massive stars may result in the hypernova branch while non-rotating and weakly rotating massive stars may the faint supernova branch. Stars belonging to the hypernova branch have large kinetic energy and ejected $^{56}$Ni mass and are implied the association with the gamma-ray bursts. On the other hand, the collapse of more massive stars in the faint supernova branch may be “observed” optically as only the disappearance of supergiants because they may not make supernova explosions. For these phenomena, a survey monitoring $\sim 10^6$ supergiants is proposed very recently (Kochanek et al. (2008). If one supergiant is confirmed to varnish from this survey, it might indicate that the black hole is formed.

1.4 Black Hole Formation

Dynamics of the black hole formation has not been studied well so far and begins to be studied recently. Fryer (1999) classified core collapse into three types. a) Stars with $M \lesssim 25M_\odot$ make explosions and produce neutron stars. b) Stars ranging $25M_\odot \lesssim M \lesssim 40M_\odot$ also result in explosions but produce black holes via fallback. c) For $M \gtrsim 40M_\odot$, the shock produced at the bounce can neither propagate out of the core nor make explosions. In any case, the core bounces once. Incidentally, the threshold between
Figure 1.4: The ejected $^{56}$Ni mass as a function of the main sequence mass of the progenitor for several supernovae from Nomoto et al. (2006).

$a)$ and $b)$ is consistent with the observational suggestion mentioned above. Sumiyoshi et al. (2006) computed fully general relativistic hydrodynamics under spherical symmetry, taking into account the reactions and transports of neutrinos in detail and confirmed class $c)$ for the collapse of a progenitor with $40M_\odot$. On the other hand, much more massive stars result in black hole formation without bounce. Sekiguchi & Shibata (2005) studied the criterion for the collapse without bounce. Their computations are fully general relativistic and they investigated the dynamics systematically, varying the initial mass and rotation. They concluded that non-rotating iron cores with a mass of $M_{\text{iron}} \gtrsim 2.2M_\odot$ collapse to black holes without bounce. However, they employed the phenomenological equation of state (EOS) and did not consider the effects of neutrinos.

There are studies also on a collapse of very massive stars in a context of the evolution of Pop III stars. As mentioned already, Pop III stars may be
very massive, $M \gtrsim 100 M_\odot$. Assuming that Pop III stars with $M \gtrsim 300 M_\odot$ are formed and evolve without mass loss, Fryer et al. (2001) and Suwa et al. (2007a) computed the collapse of rotating stars with $300 M_\odot$ under Newtonian gravity and showed that they have a weak bounce and then recollapses to a black hole immediately. Fryer et al. (2001) showed that the non-rotating star with $300 M_\odot$ collapses without bounce under spherically symmetric and fully general relativistic computation. In addition, using results of these studies, the number flux of diffuse relic neutrinos (Iocco et al. 2005) and the gravitational wave background (Suwa et al. 2007b) are also estimated. However, these studies are imperfect in the treatment of neutrino transport and the estimation of neutrino spectrum as well as in the judgment of the black hole formation.

The black hole formation by the collapse of stars in other mass ranges is also possible. Linke et al. (2001) studied on the collapse of stars with $M \gtrsim 5 \times 10^5 M_\odot$, in which neutrino emissions are taken into account together with general relativity. These stars are called supermassive stars and they become unstable due to general relativistic effects before nuclear ignition. According to their study, supermassive stars form black holes without bounce before becoming opaque to neutrinos. For the stars with mass range between $\sim 100 M_\odot$ and $\sim 260 M_\odot$, which corresponds to the iron-core mass range between $\sim 3 M_\odot$ and $\sim 30 M_\odot$, their collapse has not been studied well so far. This is because they are supposed to explode as pair-instability supernovae during the quasi-static evolutions if they are single stars. Recently, however, stars produced by stellar mergers in a young compact stellar cluster were studied in detail and their evolutionary paths are beginning to be revealed (Suzuki et al. 2007). While they did not calculate the evolutions of these stars up to the black hole formation, they may avoid the explosions as pair-instability supernovae and form a massive iron core of the above-mentioned range. Therefore, the investigation of the iron core collapses with $3-30 M_\odot$ is also meaningful, although there is no evidence to show their existence so far.

### 1.5 Intermediate Mass Black Holes

It is suggested that massive stars produced by stellar merger play important role in the formation process of SMBH residing at the center of many galaxies (e.g., Ebisuzaki et al. 2001; Portegies Zwart et al. 2006). In this section, we introduce this topics. According to the newly proposed scenario, very dense stellar clusters are initially formed in the vicinity of the galactic center ($\lesssim 10$ pc), and the massive stars with $\sim 20 M_\odot$ in them undergo runaway collisions to form very massive ($M \gtrsim 100 M_\odot$) stars before they lose most of their
mass by supernova explosions and/or pulsations. This process is confirmed by the N-body simulations (e.g., Portegies Zwart et al. 1999). Very massive stars produced by the merger will collapse to the intermediate-mass black holes (IMBH) at some stage of their evolutions. This scenario assumes that they would collapse to IMBHs when they are \( \sim 1000 M_\odot \), while it is highly uncertain at present. After that, these IMBHs merge together and finally form a SMBH.

This scenario is supported by the discovery of the ultraluminous X-ray compact sources in the galaxy M82, which indicate the existence of IMBHs. It is conceivable that similar events occur in the Milky Way Galaxy. Recent observations by Paumard et al. (2006) have revealed the existence of about 80 young massive stars within a distance of a parsec from SgrA* and some of them are identified as OB stars and their ages are about 6 \( \pm \) 2 Myr. These facts indicate that stars are actively formed in this region at present. Moreover, the IMBH candidate with \( \sim 1300 M_\odot \), IRS 13, is found in the same region (Maillard et al. 2004). Thus, SgrA* may be currently growing under this scenario. As for the event rate of the IMBH formation, based on the above-mentioned scenario and the fact that the SMBH residing in the center of our Galaxy (SgrA*) is \( \sim 3.5 \times 10^6 M_\odot \) and the age of our Galaxy is \( \sim 10 \) Gyr, a very rough estimation for the formation rate of IMBH with \( \sim 1000 M_\odot \) is \( \lesssim \) once per 1 Myr. It is, however, mentioned that this event rate may be underestimated because star formation may not be continuous but triggered by some environmental effects (e.g., the merger of galaxies).

1.6 Neutrino Astronomy

Neutrinos are helpful to examine the phenomena inside the collapsing stars. In this region, density gets much higher and photons can not escape immediately. However, neutrinos can be used to probe the dense region and, in fact, detections of neutrinos from SN1987A (e.g., Hirata et al. 1987; Bionta et al. 1987) declared that the neutrino astronomy is practical to study the physics of supernovae. Figure 1.5 is the computer print-out of the “neutrino signal” of SN1987A detected at the Kamiokande, which was a water-type Cerenkov detectors operating at the time. Future detections of supernova neutrinos will bring valuable information not only qualitatively but also quantitatively. If a supernova occurs near the Galactic center now, about 10,000 events will be detected by SuperKamiokande III (SK III), which is a descendant detector of Kamiokande and currently operating (Kotake et al. 2006). In order to confirm the mechanism of core-collapse supernovae, neutrino astronomy is essential. Moreover, the black hole formation involving neutrino emission will
Figure 1.5: SN1987A neutrino signal in the computer print-out from Koshiba (1992).
also be the target of the neutrino astronomy. On the other hand, detection of the diffuse relic neutrinos, which is a cosmological neutrino background from past core collapse supernovae and black hole formation, is also expected. While the detection of the background is severely restricted, the signal-to-noise ratio will reach a few for \( \gtrsim 10 \) years operation of SK III (Ando & Sato 2004).

Neutrino astronomy of the stellar collapse is useful not only for astrophysics but also for nuclear physics and particle physics. In fact, the existence of neutrino oscillation is convincing from the recent data of solar neutrinos as well as atmospheric neutrinos (e.g., Fukuda et al. 1999; Fukuda et al. 2001). The stellar collapse is a good laboratory for the dense matter and neutrino interactions. Their physics should reflect in the neutrino signal. In particular, we can easily predict that the black hole progenitors will undergo much higher density than the nuclear density as hadronic matter causes a deconfinement transition to quark matter (so-called QCD transition). Roughly speaking, the black hole appears when the enclosed mass of a radial coordinate \( r \) reaches
\[
M = \frac{a}{2G},
\]
where \( G \) is the gravitational constant and \( c \) is the velocity of light. The enclosed mass is associated with the density, \( \rho \), as
\[
M \sim \frac{4\pi r^3}{3} \rho.
\]
Therefore, the density at the black hole formation is expressed as
\[
\rho \sim \frac{3c^6}{32\pi G^3 M^2} \sim 1.8 \times 10^{16} \text{ g cm}^{-3} \left( \frac{M}{M_\odot} \right)^{-2}.
\]
Then matter becomes enough dense to make the QCD transition (\( \gtrsim 10^{15} \) g cm\(^{-3} \)) in the formation process of a black hole whose initial mass is less than a few times solar masses. Thus the neutrinos from black hole progenitors may give us useful information concerning such fields of physics.

### 1.7 Quark Matter in Astrophysics

It has been theoretically suggested that hadronic matter undergoes the QCD transition at high temperature and/or high density. It is also inferred that the transition occurs inside compact stars (Itoh 1970; Witten 1984; Weber 2005). Stars with a quark matter central region and a hadronic matter mantle are called hybrid stars. While the EOS for the hadron-quark mixed phase is not yet fully understood, structures and maximum masses of the hybrid stars have been well studied (e.g., Glendenning 2002). For the phase transition of a single substance, like a liquid-vapor transition of H\(_2\)O, it is simple to treat the mixed phase. On the other hand, in the hadron-quark transition, the substance is composed not only of \( u \) quarks but also of \( d \) quarks (Glendenning 1992). Incidentally, some authors use the simple Maxwell construction for the studies of hybrid stars (Marieron et al. 2004; Nicotra et al. 2006). In this scheme, Gibbs conditions for two components are not satisfied completely and
EOS’s of two phases are connected from the plot of pressure versus chemical potential.

As mentioned in the preceding section, this transition will may an important role and should be taken into account properly to investigate the stellar collapse and black hole formation. In this case, the EOS constructed in a consistent method from a lower density regime to a higher density regime with finite temperature is needed. Recently, dynamical simulations of the stellar core collapse including the transition have been done; however, their EOS is obtained at zero temperature and they adopt the Maxwell construction for mixed phase (Gentile et al. 1993; Yasutake et al. 2007). So far, the stellar collapse including the hadron-quark phase transition with a finite temperature EOS and neutrinos has not been studied. In addition, a study on the mixed phase and hot hybrid stars using Gibbs conditions for finite temperature with neutrinos exists while it does not compute the dynamics (Drago & Tambini 1999). On the other hand, the black hole formation may be a good cite to probe the properties of hot and dense matter. In particular, the trajectory of stellar collapse passes a unique regime comparing with other phenomena as shown in Figure 1.6.

1.8 Aims of this Thesis

In this thesis, we study the gravitational collapse of massive stars and their black hole formation. Here, we perform numerical simulations, under spherical symmetry, solving the general relativistic hydrodynamics and neutrino transfer equations simultaneously and treating neutrino reactions in detail.
As mentioned above, there are many types of massive stars in the universe. Thus the systematical study is needed at first. We aim to categorize a type of collapse as a function of initial iron-core mass. We also discuss the astrophysical counterparts of our models and related astronomical phenomena. To be more precise, they are stars produced by stellar mergers and the first stars in the universe (Pop III stars). We also investigate the neutrinos emitted from the collapse to suggest a way to probe into the black hole progenitors from the detections. In addition, we evaluate the flux of diffuse relic neutrino from Pop III massive stars and propose the way to explore the star formation history and the initial mass function of Pop III stars. Moreover, since the density and temperature of black hole progenitor get enough high to occur the QCD transition, we construct an EOS including the QCD transition. Since pions may also affect on the collapse, we take into account the thermal pions. Here we aim to investigate the collapse including quarks and pions to discuss the effects of them.

This thesis is organized as follows. In Chapter 2, we briefly review the neutrino radiation hydrodynamics calculated in this study. In Chapter 3, we introduce the EOS utilized in this study. The construction of the EOS including quarks and pions is also stated here. In Chapter 4, we investigate the systematic study of the stellar collapse and black hole formation. We also evaluate the neutrino event numbers from the IMBH formation at the Galactic center. More specific subjects on Pop III stars are given in Chapter 5. In Chapter 6, we explore the effects of quarks and pions performing the collapse of the realistic model obtained by evolutionary calculation of a star. Finally we conclude the thesis in Chapter 7. In Appendix A and B, we give additional discussions which may be helpful for the readers. This thesis is based on our previous studies (Nakazato et al. 2006; Nakazato et al. 2007; Nakazato et al. 2008) and work in progress. While our investigation is done under spherically symmetric geometry, various physics is involved such as, general relativity, nuclear physics and particle physics (especially for neutrinos and quarks). Moreover, the topics dealt in this thesis are also related with astrophysics and cosmology. Here we hope that this thesis serves as a fundamental study for the black hole astrophysics including even the hypernovae and gamma-ray bursts.
Chapter 2
Neutrino Radiation Hydrodynamics

2.1 Basic Equations

We compute the dynamics of spherical gravitational collapse and the neutrino transport by the general relativistic implicit Lagrangian hydrodynamics code, which solves simultaneously the neutrino Boltzmann equations (Yamada 1997; Yamada et al. 1999; Sumiyoshi et al. 2005). This code solves the evolution of the space-time adopting the metric by Misner & Sharp (1964),

\[
\begin{align*}
\text{ds}^2 = e^{2\phi(t,m)}c^2 dt^2 - e^{2\lambda(t,m)}\left(\frac{G}{c^2}\right)^2 dm^2 - r^2(t,m)(d\theta^2 + \sin^2 \theta d\phi^2)
\end{align*}
\]

(2.1)

where \(c\) and \(G\) are the velocity of light and the gravitational constant, respectively, which are taken to be unity in this chapter, and \(t, m\) and \(r\) are the time coordinate, the baryon mass coordinate and the circumference radius, respectively. The basic equations consist of the Einstein equations,

\[
G^\alpha{}^\beta = 8\pi T^\alpha{}^\beta
\]

(2.2)

obtained from the metric (2.1), and the Euler equations (note that they are not actually independent due to Bianchi identity),

\[
\nabla_\beta T^\alpha{}^\beta = 0
\]

(2.3)

with the baryon number conservation equation:

\[
\nabla_\alpha (\rho_B u^\alpha) = 0
\]

(2.4)

and evolution equation of the electron fraction (\(Y_e\)) and neutrino Boltzmann equations, which are mentioned below. Here \(G^\alpha{}^\beta\) and \(T^\alpha{}^\beta\) are the Einstein
tensor and energy-momentum tensor respectively, \( \rho_B \) is the baryon mass density and \( u^\alpha \) is 4-velocity of the matter.

The energy-momentum tensor is composed of matter part,

\[
T_m^{\alpha\beta} = [\rho_B(1 + \varepsilon) + P] u^\alpha u^\beta + P g^{\alpha\beta},
\]

where \( \varepsilon \) and \( P \) are the specific internal energy and matter pressure, and neutrino part,

\[
T_{\nu}^{\alpha\beta} = \frac{1}{h^3} \int f_{\nu} p^\alpha p^\beta \sqrt{-g} \frac{dp^m dp^\theta dp^\phi}{p_t},
\]

\[
\sqrt{-g} = e^{\phi + \lambda r^2 \sin^2 \theta},
\]

for each species. Here \( p^\alpha \) and \( f_{\nu} \) are neutrino 4-momentum and neutrino distribution function respectively and \( h \) is Planck constant. We can rewrite equation (2.6a) assuming \( f_{\nu} \) as a function of \( (t, m, p, \mu) \), where \( p \) is neutrino momentum in the comoving fluid frame and \( \mu \) is cosine of the angle between the neutrino momentum and the radial direction in the fluid frame, as below:

\[
T_{\nu}^{tt} = e^{-2\phi} u_{\nu},
\]

\[
T_{\nu}^{tm} = e^{-\phi - \lambda} F_{\nu},
\]

\[
T_{\nu}^{mm} = e^{-2\lambda} P_{\nu},
\]

\[
r^2 T_{\nu}^{\theta\theta} = r^2 \sin^2 \theta T_{\nu}^{\phi\phi} = \frac{u_{\nu} - p_{\nu}}{2},
\]

where,

\[
u_{\nu} \equiv \frac{1}{h^3} \int p f_{\nu} d^3 p,
\]

\[
F_{\nu} \equiv \frac{1}{h^3} \int p \mu f_{\nu} d^3 p,
\]

\[
P_{\nu} \equiv \frac{1}{h^3} \int p \mu^2 f_{\nu} d^3 p.
\]

We also solve Boltzmann equation,

\[
\frac{df_{\nu}}{d\lambda'} = (\frac{df_{\nu}}{d\lambda'})_{\text{coll}},
\]

where differential by \( \lambda' \) means differential along the neutrino trajectory which is equivalent to geodesics when we neglect the mass of neutrino. The right
hand side of equation \( (2.9) \) means the change of \( f_\nu \) by reaction, scattering and pair creation, which are listed below. We can calculate this team as
\[
\left( \frac{df_\nu}{d\lambda'} \right)_{\text{coll}} = p \left( \frac{df_\nu}{dt_p} \right)_{\text{coll}}, \tag{2.10}
\]
where \( t_p \) is proper time on the comoving fluid frame. On the other hand, we can rewrite the left hand side of equation \( (2.9) \) as below:
\[
\frac{df_\nu}{d\lambda'} = \frac{dx^\alpha}{d\lambda'} \frac{\partial f_\nu}{\partial x^\alpha} + \frac{dp^a}{d\lambda'} \frac{\partial f_\nu}{\partial p^a} = \frac{p^\alpha}{\partial x^\alpha} - \Gamma^a_{bc} p^b \frac{\partial f_\nu}{\partial p^a}; \tag{2.11}
\]
where \( \Gamma^a_{bc} \) is the Cristoffel symbol. Note that \( p^0 \) is not independent of \( p^1, p^2, p^3 \). Moreover, neutrino reactions cause the fluctuation of \( Y_e \). Thus evolution equation of \( Y_e \) is written as,
\[
e^{-\phi} \frac{\partial Y_e}{\partial t} = -\frac{m_u}{\rho_b h^3} \int \left( \frac{df_\nu}{d\lambda'} \right)_{\text{coll}} \frac{d^3p}{p}, \tag{2.12}
\]
where \( m_u \) is the atomic mass unit.

It is noted that this time-slicing allows us to follow the dynamics with no difficulty until a black hole formation. However, the original code is meant to be employed for the supernova simulation, in which gravity is not so strong. As the black hole formation is approached and gravity becomes strong, we find it better to use the following form for the matter contribution to the energy equation,
\[
e^{-\phi} \left( \frac{\partial \varepsilon}{\partial t} \right)_m = -\frac{p}{\Gamma} \frac{\partial}{\partial m} (4\pi r^2 U), \tag{2.13}
\]
where \( \varepsilon \) and \( p \) are the specific internal energy and matter pressure, and \( U \) and \( \Gamma \) are the radial fluid velocity and general relativistic Lorentz factor defined as
\[
U = e^{-\phi} \frac{\partial r}{\partial t}, \tag{2.14a}
\]
\[
\Gamma = e^{-\lambda} \frac{\partial r}{\partial m}, \tag{2.14b}
\]
respectively. This form of energy equation is used only when \( \Gamma < 0.7 \). We use 127 radial mesh points and set a fixed boundary condition at the outermost grid point. The radius of the outer boundary is chosen to be large enough not to affect the results.
2.2 Criterion for Apparent Horizon

We can utilize the apparent horizon to judge the black hole formation and it is the outermost trapped surface. The trapped surface is the surface where both the ingoing and outgoing null geodesics have a negative expansion. The set of the trapped surfaces is called a trapped region. In the Misner-Sharp metric, the condition for the trapped region is expressed as

\[ U + \Gamma < 0, \]

which is equivalent to

\[ r < r_g \equiv 2\tilde{m}, \]

where \( \tilde{m} \) denotes the gravitational mass and it is related to \( U, \Gamma \) and \( r \) as

\[ \Gamma^2 = 1 + U^2 - \frac{2\tilde{m}}{r}. \]

Since it is proved that the apparent horizon is always located inside the event horizon (e.g., Wald 1984), the existence of apparent horizon is a sufficient condition for the black hole formation.

2.3 Treatment of Neutrinos

We compute the neutrino distribution functions for 4 species of neutrino (\( \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu \)) on a discretized energy- and angular-grid points by solving the neutrino Boltzmann equations. In our simulations, the energy space is discretized to 12 mesh points and the angular space is discretized to 4 mesh points. As for the convergence, see in Appendix A. We consider 4 species of neutrino, \( \nu_e, \bar{\nu}_e, \nu_\mu \) and \( \bar{\nu}_\mu \), assuming that \( \nu_\tau \) and \( \bar{\nu}_\tau \) are the same as \( \nu_\mu \) and \( \bar{\nu}_\mu \), respectively. For the collision term in equation (2.9), we calculate the scattering kernels explicitly in terms of angles and energies of incoming and outgoing neutrinos (Mezzacappa & Bruenn 1993). We take into account the following neutrino reactions:

[1] electron-type neutrino absorption on neutrons and inverse,

\[ \nu_e + n \longleftrightarrow e + p, \]  

(2.18)

[2] electron-type anti-neutrino absorption on protons and inverse,

\[ \bar{\nu}_e + p \longleftrightarrow e^+ + n, \]  

(2.19)

[3] neutrino scattering on nucleons,

\[ \nu + N \longleftrightarrow \nu + N, \]  

(2.20)
[4] neutrino scattering on electrons,

\[ \nu + e \leftrightarrow \nu + e, \]  

(2.21)

[5] electron-type neutrino absorption on nuclei,

\[ \nu_e + A \leftrightarrow A + e^-, \]  

(2.22)

[6] neutrino coherent scattering on nuclei,

\[ \nu + A \leftrightarrow \nu + A, \]  

(2.23)

[7] electron-positron pair annihilation and creation,

\[ e^- + e^+ \leftrightarrow \nu + \bar{\nu}, \]  

(2.24)

[8] plasmon decay and creation,

\[ \gamma^* \leftrightarrow \nu + \bar{\nu}, \]  

(2.25)

[9] neutrino bremsstrahlung,

\[ N + N' \leftrightarrow N + N' + \nu + \bar{\nu}. \]  

(2.26)

Chapter 3

Equation of State

3.1 Relativistic EOS of Nuclear Matter

In this study, we use EOS by Shen et al (1998a, 1998b), which is derived in the relativistic mean field theory with the Lagrangian

\[
\mathcal{L}_{RMF} = \bar{\psi} [i \gamma_{\mu} \partial^{\mu} - M - g_\sigma \sigma - g_\omega \gamma_{\mu} \omega^{\mu} - g_\rho \gamma_{\mu} \tau_a p^{a\mu}] \psi \\
+ \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
- \frac{1}{4} W_{\mu \nu} W^{\mu \nu} + \frac{1}{2} m_\omega^2 \omega_{\mu} \omega^{\mu} + \frac{1}{4} c_3 (\omega_{\mu} \omega^{\mu})^2 \\
- \frac{1}{4} R_{\mu \nu}^a R^{a\mu \nu} + \frac{1}{2} m_\rho^2 \rho_{\mu} \rho^{a\mu},
\]  

(3.1)

where

\[
W^{\mu \nu} = \partial^{\mu} \omega^{\nu} - \partial^{\nu} \omega^{\mu},
\]  

(3.2a)

\[
R^{a\mu \nu} = \partial^{\mu} \rho^{a \nu} - \partial^{\nu} \rho^{a \mu} + g_\rho \epsilon^{abc} \rho^{b \mu} \rho^{c \nu}.
\]  

(3.2b)

In these equations, \(\psi, \sigma, \omega^{\mu}\) and \(\rho^{a \mu}\) denote fields of a \(SU(2)\) baryon with mass \(M\) (both for proton and neutron), \(\sigma\) meson with mass \(m_\sigma\), \(\omega\) meson with mass \(m_\omega\) and \(\rho\) meson with mass \(m_\rho\), respectively. Here, \(g_\sigma, g_\omega\) and \(g_\rho\) are the coupling constants for the interactions between mesons and nucleons, \(g_2\) and \(g_3\) are the self-coupling constants for \(\sigma\) meson field and \(c_3\) is the self-coupling constant for \(\omega\) meson field. It is known that the nonlinear \(\sigma\) terms are essential to quantitatively reproduce the properties of nuclei and a reasonable value for the incompressibility (Sugahara & Toki, 1994).

For \(g_\sigma, g_\omega, g_\rho, g_2, g_3\) and \(c_3\), Shen EOS adopted the parameter set named TM1. With the parameter set TM1, the symmetry energy is 36.9 MeV.
and the incompressibility is 281 MeV, which is larger than that in the non-relativistic framework (e.g., Lattimer & Swesty 1991). Therefore the resultant EOS becomes stiff and its maximum mass of the neutron star is large as \( \approx 2.2M_\odot \) (Sumiyoshi et al. 1995a). The table of this EOS covers a wide baryon-mass density range, \( \rho_B = 10^{5.1} - 10^{15.4} \text{ g cm}^{-3} \), electron fraction range, \( Y_e = 0 - 0.56 \), and temperature range, \( T = 0 - 100 \text{ MeV} \). An inhomogeneity of the matter is also taken into account using the Thomas-Fermi approximation because heavy nuclei appear in \( T \lesssim 15 \text{ MeV} \) and densities below the nuclear matter density.

### 3.2 Setups for Hadron-Quark Mixed EOS

As mentioned already, the hadron-quark transition (QCD transition) will occur in the formation process of a black hole. In this section, we construct the EOS for hadron, quark and their mixture as a function of baryon mass density \( \rho_B \), electron fraction \( Y_e \) and temperature \( T \). The EOS calculated in this study covers \( 10^{5.1} \text{ g cm}^{-3} \leq \rho_B \leq 10^{17} \text{ g cm}^{-3} \), \( 0 \leq Y_e \leq 0.56 \) and \( 0 \text{ MeV} \leq T \leq 400 \text{ MeV} \). The lower limit of \( \rho_B \) and the range of \( Y_e \) are the same as those of the Shen EOS, which is adopted for the pure hadronic matter in our model. We assume that the pure hadronic matter exists under the transition density and the pure quark matter exists for a higher density than that of the end point of the mixed phase. In the following, after we introduce the EOS for pure hadronic matter and pure quark matter, we give the formulations of the EOS for mixed matter. Finally, we also describe the treatment of muons.

#### 3.2.1 EOS for Pure Hadronic Matter

We adopt the Shen EOS in § 3.1 for the pure hadronic matter. It is noted that the density where the hadron-quark transition occurs is higher than the nuclear matter density for \( T \lesssim 15 \text{ MeV} \) in our model. Thus the hadronic matter at the transition point is composed of dissociated protons and neutrons. It is also noted that we extend the EOS to \( T > 100 \text{ MeV} \) by a method fully consistent with the Shen EOS performing the calculations based on the relativistic mean field theory.

We add the effects of charged pions, \( \pi^\pm \), and neutral pions, \( \pi^0 \), to the EOS for the pure hadronic matter. The original Shen EOS is based on the mean field theory without the pions. It is noted that the pion field vanishes in the mean field approximation because it is a pseudoscalar particle (Glendenning et al. 1983a). In the realistic dense matter, pions may exist
though large uncertainties still exist. Thus we examine their qualitative effects by treating them in the minimum model. Here we employ the method in Glendenning et al. (1983a), Glendenning et al. (1983b) and Glendenning (1985) under the assumption that an effective mass of pions is equal to their rest mass in vacuum. Strictly speaking, pions feel an repulsive potential in the nucleons and their effective mass gets larger than that in vacuum. In this case, the pion population is suppressed. Thus our assumption corresponds to one extreme case and the situation without pions does the other extreme case (Glendenning et al. 1983a; Glendenning et al. 1983b). The realistic condition should be between these two cases.

The pions are handled as an ideal boson gas and the chemical potentials of the neutral and charged pions are determined as follows. The chemical potential of the neutral pion is always \( \mu_{\pi^0} = 0 \) because it is a self-conjugate particle and created by the pair process of two photons. As for the chemical potentials of the charged pions, we set them as \( \mu_{\pi^-} = \mu_n - \mu_p \) and \( \mu_{\pi^+} = -\mu_{\pi^-} \), where \( \mu_p \) and \( \mu_n \) are the chemical potentials of proton and neutron, respectively. When the charge chemical potential, \( \mu_n - \mu_p \), exceeds the rest mass of charged pions, \( m_{\pi^\pm} \), at high densities, the charged pions are condensed and the charge chemical potential decreases to their rest mass. In fact, there is such a regime in the original Shen EOS table. It is well known that this threshold is sensitive to the density dependence of the symmetry energy. We remark that the density dependence is strong in the relativistic nuclear many body frameworks such as the relativistic mean field theory for Shen EOS, as compared with the non-relativistic counter part (Sumiyoshi et al. 1995b).

In the following, we replace the original Shen EOS given as a function of baryon mass density \( \rho_B \), proton fraction \( Y_p \) and temperature \( T \) with our hadronic EOS given as a function of \( \rho_B, Y_C \) and \( T \). \( Y_C \) is a charge fraction and defined as

\[
Y_C = \frac{n_p - n_{\pi^-}}{n_B},
\]

where \( n_B \) and \( n_p \) are the baryon number density and the number density of protons, respectively. \( n_{\pi^-} \) is a net number density of the charged pions, which is the difference of the number density of \( \pi^- \) to that of \( \pi^+ \), and calculated from the Bose-Einstein distribution function:

\[
n_{\pi^-} = \frac{1}{h^3} \int_0^\infty \left[ \frac{1}{\exp \left( \sqrt{m_{\pi^\pm}^2 c^4 + p^2 c^2} - \mu_{\pi^-} \right)/(k_B T) - 1} - \frac{1}{\exp \left( \sqrt{m_{\pi^\pm}^2 c^4 + p^2 c^2} + \mu_{\pi^-} \right)/(k_B T) - 1} \right] 4\pi p^2 dp,
\]

where \( h, k_B \) and \( c \) are the Planck constant, the Boltzmann constant and the
velocity of light, respectively. When the pions are not condensed, we find \( \mu_{\pi^-} \) and the corresponding state of the nucleons in the Shen EOS for given \( \rho_B, Y_C \), and \( T \). In this process, we determine the relations of \( n_p, \mu_p \), and \( \mu_n \) from the Shen EOS. Using \( \mu_{\pi^-} \) found above, the pressure and energy density of the pions are calculated.

On the other hand, in case of the pion condensation, we fix the chemical potential as \( \mu_{\pi^-} = m_{\pi^-} + c^2 \) and calculate the number density of the “thermal” pions, \( n_{\pi^-}^{th} \), in equation (3.3). The pressure and energy density of “thermal” pions are given from the Bose-Einstein distribution function as in the case without the pion condensation. At the same time, we can determine the EOS of nucleons for given \( \rho_B \) and \( T \) using the Shen EOS under the condition of \( \mu_n - \mu_p = m_{\pi^-} + c^2 (= \mu_{\pi^-}) \). Having \( Y_p \) and \( n_p \) fixed, we can get the net number density of the pions, \( n_{\pi^-} \), for given \( Y_C \) using equation (3.3). Here we set the number density of the “condensed” pions as \( n_{\pi^-}^{cond} = n_{\pi^-} - n_{\pi^-}^{th} \). These condensed pions contribute not to the pressure but to the energy density by their rest masses. We can determine the EOS including condensed pions approximately.

Incidentally, when the electron-type neutrinos are trapped and in equilibrium with other particles, their chemical potential should be given as equation (3.10a), which is expressed later. For high density regime where the hadron-quark transition occurs, neutrinos are fully trapped and we show the results including trapped neutrinos in § 3.3.1. On the other hand, since neutrinos are not trapped at least for the onset of gravitational collapse, we follow the time evolution of neutrino distributions through neutrino reactions for numerical simulations in § 6.2. In addition to the quark degree of freedom, hyperons and kaons also may be important in the high density regime. In particular, since hyperons will appear before the hadron-quark phase transition (Glendenning, 1985; Balberg et al. 1999; Ishizuka et al. 2008), we are investigating their effects in future work.

### 3.2.2 EOS for Pure Quark Matter

We adopt the MIT bag model (Chodos et al. 1974) as the pure quark matter. This is a phenomenological model which describes the nature of the confinement and the asymptotic freedom of quarks. In this model, free quarks are confined in the “bag” and this “bag” has a positive potential energy per unit volume (Chodos et al. 1974). Thus the thermodynamical potential is expressed as \( \Omega_Q = \Omega_0 + BV^{\frac{1}{3}} \) where \( V \) and \( \Omega_0 \) are the volume of the “bag”
and the thermodynamical potential of free quarks as ideal fermions, respectively. The bag constant, $B$, is a parameter characterizing this model, and we investigate its dependence later. From thermodynamical relations, we can calculate the number density $n_Q$, pressure $P_Q$ and energy density $\varepsilon_Q$ for the quark matter with the temperature $T$ as

$$n_Q = \sum_f \frac{g}{h^3} \int_0^\infty \left( F_f^+ (p) - F_f^- (p) \right) \frac{4 \pi p^2 dp}{}, \quad (3.5a)$$

$$P_Q = \sum_f \frac{g}{h^3} \int_0^\infty \frac{p^2 c^2}{3 \sqrt{m_f^2 c^2 + p^2 c^2}} \left( F_f^+ (p) + F_f^- (p) \right) \frac{4 \pi p^2 dp}{}, -B(3.5b)$$

$$\varepsilon_Q = \sum_f \frac{g}{h^3} \int_0^\infty \sqrt{m_f^2 c^2 + p^2 c^2} \left( F_f^+ (p) + F_f^- (p) \right) \frac{4 \pi p^2 dp}{}, + B, \quad (3.5c)$$

where $F_f^+ (p)$ and $F_f^- (p)$ represent the Fermi-Dirac distribution functions for particle and antiparticle, respectively, and they are expressed as

$$F_f^\pm (p) = \frac{1}{\left\{ \exp \left( \sqrt{m_f^2 c^2 + p^2 c^2} \mp \mu_f / (k_B T) \right) \right\} + 1}. \quad (3.6)$$

The subscript $f$ denotes the flavor of quarks, and we take into account three flavors, namely, $u$-, $d$- and $s$ quarks. The statistical weight is $g = 2 \times 3$. $m_f$ and $\mu_f$ are the mass and the chemical potential of $f$ quark, respectively, and we adopt $m_u c^2 = 2.5$ MeV, $m_d c^2 = 5$ MeV and $m_s c^2 = 100$ MeV in this study (Yao et al. 2006).

As mentioned already, the EOS for the pure hadronic matter is given as the function of the baryon mass density $\rho_B$, the charge fraction $Y_C$ and the temperature $T$. We can rewrite the EOS of the MIT bag model as the function of these three independent variables. For convenience, we define the “baryon” number density and the “baryon” mass density of the quark matter as

$$n_{B,Q} = \frac{n_Q}{3}, \quad (3.7a)$$

$$\rho_{B,Q} = m_{\text{unit}} \frac{n_Q}{3}, \quad (3.7b)$$

where the value of the atomic mass unit $m_{\text{unit}} = 931.49432$ MeV is used as in the Shen EOS. Since the electric charges of $u$-, $d$- and $s$ quarks are $\frac{2}{3}$, $-\frac{1}{3}$ and $-\frac{1}{3}$, however, the validity is not guaranteed for the densities in which we are interested because the perturbative approach is valid only in the high energy limit. Hence, we do not take into account these effects in this study.
CHAPTER 3. EQUATION OF STATE

and $-\frac{1}{3}$ of that of a proton, respectively, the charge fraction of the quark matter is as follows:

$$Y_{C,Q} = \frac{2n_u - n_d - n_s}{n_u + n_d + n_s} = \frac{2n_u - n_d - n_s}{n_Q},$$

(3.8)

where $n_f$ is a number density of $f$ quarks. When the $\beta$ equilibrium is satisfied, $\mu_d$ is equal to $\mu_s$, which is expressed later as equation (3.10d). Thus we can determine $\mu_u$, $\mu_d$ and $\mu_s$ for given $\rho_{B,Q}$, $Y_{C,Q}$ and $T$, and we can express $P_Q$ and $\varepsilon_Q$ as functions of $\rho_{B,Q}$, $Y_{C,Q}$ and $T$. We can also calculate other variables, such as the entropy density $s_Q = \frac{P_Q + \varepsilon_Q}{T} + \frac{1}{T} \sum_f n_f \mu_f$, the Helmholtz free energy per unit volume $\mathcal{F}_Q = \varepsilon_Q - Ts_Q$ and the Gibbs free energy per unit volume $G_Q = \mathcal{F}_Q + P_Q$. We will use these variables later.

### 3.2.3 EOS for Hadron-Quark Mixed Phase

Following Glendenning (1992), we first show the conditions of the equilibrium between phases in the heat bath with the temperature $T$. Here, we deal with hadronic matter, quark matter, electrons, electron type neutrinos and mu-type leptons, whose treatment is stated later in §3.2.4. We assume that the equilibrium is achieved not only by the strong interactions but also by the weak interactions. This is because only $u$- and $d$ quarks are deconfined from protons and neutrons and $s$ quarks are created by the weak interactions. Moreover, we assume that the neutrinos are completely trapped owing to high density. In other words, the diffusion time scale is much longer than the reaction time scale. Thus the following reactions are in chemical equilibrium:

$$p + e^- \leftrightarrow n + \nu_e,$$

(3.9a)

$$n \leftrightarrow u + 2d,$$

(3.9b)

$$p \leftrightarrow 2u + d,$$

(3.9c)

$$d \leftrightarrow u + e^- + \bar{\nu}_e,$$

(3.9d)

$$s \leftrightarrow u + e^- + \bar{\nu}_e,$$

(3.9e)

$$u + d \leftrightarrow u + s,$$

(3.9f)

and the relations of the chemical potentials are given as

$$\mu_p + \mu_e = \mu_n + \mu_\nu,$$

(3.10a)

$$\mu_n = \mu_u + 2\mu_d,$$

(3.10b)

$$\mu_p = 2\mu_u + \mu_d,$$

(3.10c)

$$\mu_d = \mu_s.$$

(3.10d)
It is noted that electrons reside in both phases and the chemical potential for each phase coincides with each other. This is also true for neutrinos. It goes without saying that two phases are also in mechanical equilibrium. For simplicity, we ignore the surface tension and the screening of the charged particles though they may affect the EOS (Endo et al. 2006; Maruyama et al. 2007). Thus we require the condition,

\[ P_H = P_Q, \]  

where \( P_H \) and \( P_Q \) are the pressures of hadronic and quark phases, respectively. It is noted that we do not need to take into account the contribution of leptons in equation (3.11), because their chemical potentials and pressures are the same for each phase.

Next, we relate the independent variables in each phase to the average values using a volume fraction of the quark phase, \( \chi \). For instance, a pure hadronic phase corresponds to \( \chi = 0 \) and a pure quark phase to \( \chi = 1 \). Then the baryon mass density of the mixed phase, \( \rho_B \), is written as

\[ \rho_B = (1 - \chi)\rho_{B,H} + \chi\rho_{B,Q}, \]  

where \( \rho_{B,H} \) is the baryon mass density of the hadronic phase and \( \rho_{B,Q} \) is that of the quark phase and given in equation (3.7b). We note that, the charge fractions of both phases, \( Y_{C,H} \) and \( Y_{C,Q} \), do not necessarily coincide with each other. Since the mixed phase is charge neutral as a whole, we can relate these values with the electron fraction of the mixed phase, \( Y_e \), as

\[ Y_e \rho_B = (1 - \chi)Y_{C,H} \rho_{B,H} + \chi Y_{C,Q} \rho_{B,Q}. \]  

It is noted that the charge neutrality is not required for each phase independently (Glendenning, 1992). This fact gives an essential difference from the phase transition of a single substance like a liquid-vapor transition of H$_2$O. For instance, the pressure of the hadron-quark mixed phase is not constant in an isothermal process. Now the construction of the EOS for hadron-quark mixed phase is reduced to the determination of the volume fraction, \( \chi \), for given \( \rho_B, Y_e \) and \( T \). Mathematically, it is equivalent to solving the system of 5 equations (3.10b, 3.10c, 3.11, 3.12, 3.13) for 5 variables, namely, \( \chi, \rho_{B,H}, Y_{C,H}, \rho_{B,Q} \) and \( Y_{C,Q} \). Here \( \rho_B \) and \( Y_e \) are given, \( \mu_n, \mu_p \) and \( P_H \) are the functions of \( \rho_{B,H} \) and \( Y_{C,H} \), and \( \mu_u, \mu_d \) and \( P_Q \) are the functions of \( \rho_{B,Q} \) and \( Y_{C,Q} \).

### 3.2.4 Treatment of Muons

The treatment of muons and their anti-particles is important because their compositions affect the EOS. Here, we examine two situations. The first one
corresponds to the EOS for stellar core collapse. Here, we call it supernova (SN) matter. In the case of an ordinary supernova, it is known that not only the net muon fraction but also the muon and anti-muon fractions themselves are minor because the rest mass of the muon is large \( m_\mu = 105.66 \text{ MeV} \). Then, they are omitted in simulations for the ordinary supernova. On the other hand, we may not be able to neglect their effects for EOS in case of black hole formation, because the density and temperature would be higher than those for the ordinary supernova (e.g., Sumiyoshi et al. 2005).

During the core collapse and bounce, muons behave as follows. Since the charged current reactions for mu-type leptons are not efficient before neutrino trapping, mu-type neutrinos \( \nu_\mu \) and their anti-particles \( \bar{\nu}_\mu \) are produced mainly by pair processes, and their reactions are almost identical. Thus one can safely posit that the net muon fraction is zero during the initial collapse before neutrino trapping. When neutrinos are fully trapped, the charged current reactions may become efficient. Therefore, muons and mu-type neutrinos are in chemical equilibrium. \( \nu_\mu \) and \( \bar{\nu}_\mu \) cannot be regarded as identical, and the net muon fraction can become non-zero.

In our EOS, in order to add the effects of muon and anti-muon pairs in a tractable manner, we do not assume chemical equilibrium for muon type leptons.\(^2\) but suppose that the net muon fraction is zero (i.e., \( \mu_\mu = 0 \)). The approximation \( \mu_\mu = 0 \) is reasonable because the pair population dominates in higher temperature regime, where we cannot neglect the effects of muon-type leptons for EOS in the black hole formation. Incidentally, this assumption is consistent with the setups of our simulations stated in Chapter 2, which does not take into account the muon-charged current reactions. In order to treat the muon-charged current reactions, one has to solve the neutrino-transfer for 6 species, which requires formidable computing resources. It is also necessary to prepare the EOS table as a function of not only \( Y_e \) but also \( Y_\mu \), which makes the setups further complicated. The muon-charged current reactions should be included in future to assess their effects.

The second case of the treatment of muons is for so-called neutron star (NS) matter. In this case, the neutrino less \( \beta \) equilibrium at zero temperature is achieved and the chemical potential of the muons, \( \mu_\mu \), is the same as that of electrons. Thus the relations of the chemical potentials become as follows:

\[
\begin{align*}
\mu_e & = \mu_n - \mu_p, \quad (3.14a) \\
\mu_\mu & = \mu_e. \quad (3.14b)
\end{align*}
\]

\(^2\) We remark that the \( \beta \) equilibrium assumed in previous sections is for the electron-type leptons. This assumption is not inconsistent with the treatment of muons.
CHAPTER 3. EQUATION OF STATE

Moreover, the charge neutrality condition (3.13) is modified to

\[(Y_e + Y_\mu)\rho_B = (1 - \chi)Y_{C,H}\rho_{B,H} + \chi Y_{C,Q}\rho_{B,Q},\]  

(3.15)

where \(Y_\mu\) is a muon fraction.

3.3 Results of Hadron-Quark Mixed EOS

In this section, we show the features of the EOS described in the preceding section. We choose a mixed EOS with the hadronic matter including pion and the quark matter of the bag constant \(B = 250\) MeV fm\(^{-3}\) \((B^{1/4} = 209\) MeV\), as a reference model. This value is adopted in another recent study on the hadron-quark phase transition (Wang et al. 2007). Here we discuss the properties of EOS with the applications to the maximum mass of compact stars. We mainly show the results of the reference model for the NS matter and the SN matter. Incidentally, results of the model without pions are also shown, and the dependence on the bag constant is studied for several topics.

3.3.1 Assessments of EOS

First of all, we examine the composition of our EOS for NS matter and SN matter to compare with other EOS’s in the previous studies using similar schemes (Glendenning 1985; Glendenning 1992; Drago & Tambini 1999; Nicotra et al. 2006). For SN matter, we show the cases of \(T = 50\) MeV with the electron type lepton fraction \(Y_l = 0.1\), where \(Y_l\) is defined as the sum of the electron fraction, \(Y_e\), and the electron-type neutrino fraction, \(Y_\nu_e\). Note that when the neutrinos are fully trapped, \(Y_l\) is conserved for each fluid element in the stellar core. In Figure 3.1, we show the particle fractions, \(Y_i \equiv \frac{n_i}{n_B}\), where \(n_i\) represents the number density of the particle \(i\), for the reference models. When pions begin to condensate in the NS matter, the proton fraction increases and the fractions of electron and muon decrease (Glendenning 1985). At the onset of the hadron-quark phase transition, \(d\) quarks have larger population than those of \(u\)- and \(s\) quarks not only for the NS matter (Glendenning 1992) but also for the SN matter (Drago & Tambini 1999). In the pure quark phase, the number ratio of \(u\)-, \(d\)- and \(s\) quark is nearly equi-partition and the relation, \(Y_s < Y_u < Y_d\) holds for the NS matter. For the SN matter with neutrino trapping, \(u\) quarks have larger population than those of \(d\)- and \(s\) quarks (Drago & Tambini 1999; Nicotra et al. 2006). All of these features are consistent with the previous results.

In Figure 3.2 we compare the EOS for the reference model with those for pure hadronic and quark matters, where the bag constant \(B = 250\) MeV fm\(^{-3}\).
Figure 3.1: Particle fractions for the reference model of NS matter (left) and SN matter with $T = 50$ MeV and $Y_l = 0.1$ (right). The particle fraction, $Y_i$, is defined as $\frac{n_i}{n_B}$, where $n_i$ represents the number density of the particle $i$.

This figure illustrates both for NS matter and SN matter with $T = 20, 50$ and $120$ MeV and $Y_l = 0.1$. As mentioned already, the pressure of the hadron-quark mixed phase is not constant in an isothermal process (Glen- denning 1992), which is well presented in our models. In addition, this trend is shown also in the other previous work (Burgio et al. 2002) while the bag constant is different from ours. The internal energy, which is the difference of the energy density with respect to $\rho B c^2$, of the mixed phase is larger than that of the hadronic phase for high temperature regime ($T \geq 50$ MeV). This is not unphysical because we should compare not the internal energies but the free energies. In the left panels of Figure 3.3 we show the Helmholtz free energies per baryon as a function of the specific volume. If the EOS is thermodynamically stable, this function is convex downward. This feature is fulfilled for our models. In the right panels of Figure 3.3 on the other hand, we show the Gibbs free energies per baryon as a function of the pressure. We can recognize that the free energies of the mixed phase are always lower than those of the pure hadronic and quark phases.

For the phase transition of matter with two components, it is known that the mixed phase should satisfy not only the condition of mechanical stability, as verified above, but also the stability with respect to diffusion (Prigogine
Figure 3.2: EOS’s for mixed matter (solid lines), pure hadronic matter (long-dashed lines) and pure quark matter (short-dashed lines). In all panels, the bag constant is chosen as $B = 250$ MeV fm$^{-3}$ for the quark model. The left and right panels show the pressure and the internal energy, respectively, as a function of the density. In both panels, the upper left, the upper right, the lower left and the lower right plots correspond to the reference models of NS matter, SN matter with $T = 20$, 50 and 120 MeV, respectively. The electron type lepton fraction, $Y_e$, is fixed to 0.1 for all models of SN matter.

(& Defay 1954). In our model, the condition is expressed as

$$\left( \frac{\partial \mu_n}{\partial X_p} \right)_{P,T} \leq 0,$$

where $X_p$ is a “total” proton fraction and defined as

$$X_p = \frac{1}{n_B} \left( n_p + \frac{2n_u - n_d - n_s}{3} \right).$$

This feature is shown for our results in Figure 3.4 and satisfied also in other ranges of the parameters. The reference model of the SN matter with $Y_e = 0.1$ for fixed pressure and temperature are indicated in this figure. From these analyses, we can confirm that our EOS is thermodynamically stable.

In Figure 3.5 we show the temperature dependences of the transition density and the critical baryon chemical potential for our EOS of the SN matter with $Y_e = 0.1$ and 0.3. These phase diagrams are given not only for
the reference model but also for a model without pions while the bag constant of both models is 250 MeV fm$^{-3}$. The baryon chemical potential in these figures, $\mu_B$, is the same as the neutron chemical potential in the preceding sections, $\mu_n$. We can see that the reference model has larger transition density and baryon chemical potential than those of the model without pions. This is because pions make the EOS softer; in other words, pressure gets lower for a fixed baryon number density. Thus the condition of equilibrium (3.11) is satisfied for larger density. Incidentally, the phase diagram has been studied in Drago & Tambini (1999) while its EOS for hadronic matter does not include pions. The color dielectric model is used for the quark matter model whereas we use the MIT bag model. In Drago & Tambini (1999), the transition density gets lower at higher temperature, which is the same as our result, however, its transition density is lower than ours, especially at finite temperature. For instance, the transition density is $\sim 10^{14}$ g cm$^{-3}$ at

Figure 3.3: EOS’s for mixed matter (solid lines), pure hadronic matter (long-dashed lines) and pure quark matter (short-dashed lines). In all panels, the bag constant is chosen as $B = 250$ MeV fm$^{-3}$ for the quark model. The upper left and upper right panels show the Helmholtz free energies per baryon ($F$) as a function of a specific volume of the system and the Gibbs free energies per baryon ($G$) as a function of the pressure, respectively, for the reference model of NS matter. The lower two panels are the same as upper two panels but for the reference model of SN matter with $T = 50$ MeV and $Y_l = 0.1$. 

\[ F, G \text{ vs. Volume (fm}^3\text{)} \]
\[ F, G \text{ vs. Pressure (MeV/fm}^3\text{)} \]
Figure 3.4: Profiles of neutron chemical potential, $\mu_n$, as a function of “total” proton fraction, $X_p$, for fixed pressure $P = 220$ MeV fm$^{-3}$ and temperatures $T = 50$ MeV (solid line) and $63$ MeV (dashed line). This is a result for the reference model of SN matter with $Y_l = 0.1$.

$T \sim 30$ MeV.

It is suggested that the transition line has an end point in the high temperature regime (Asakawa & Yazaki 1989). The nature of the transition changes at this point, while the exact phase diagram is not well known. This is a so-called critical point. The temperature of the critical point, $T_c$, is investigated experimentally by heavy-ion collisions (Adams et al. 2005; Adcox et al. 2005) and theoretically by lattice QCD calculations (Bernard et al. 2005; Cheng et al. 2006; Aoki et al. 2006a, 2006b). From these studies, it may be in the range, $150$ MeV $\leq T_c \leq 200$ MeV. Above the critical temperature, the quark phase may be the most stable state for any densities. It is also suggested that the baryon chemical potential of the critical point is much smaller than the typical hadronic scale, $\mu_B \lesssim 40$ MeV. Although our model cannot describe the critical point in detail, the critical baryon chemical potential drops dramatically with the temperature in the regime $T \gtrsim 100$ MeV. For much higher temperature regime, the quark matter is more stable than the hadronic matter even at zero density. As shown in Figure 3.6, for instance, the hadron-quark phase transition occurs at $T = 150$ MeV and the quark phase is always the most stable state at $T = 200$ MeV for the model with $B = 250$ MeV fm$^{-3}$. Therefore, the plots for $T \gtrsim 150$ MeV are not given in Figure 3.5. In our model, the
Figure 3.5: Phase diagrams of our EOS of the SN matter with $Y_l = 0.1$ and 0.3 for $T < 150$ MeV. Solid lines represent boundaries of hadronic matter and mixed matter and dashed lines do those of mixed matter and quark matter. For the upper plots in each panel, an EOS with pions is used while an EOS without pions is used for the lower plots in each panel. $B = 250$ MeV fm$^{-3}$ is chosen for the bag constant for all panels.
3.3.2 Maximum Mass of the Hybrid Stars

We examine the maximum mass of hybrid stars constructed by our EOS for NS matter. In the following, we denote the EOS’s without pions and quarks (the original Shen EOS in § 3.1), without pions but with quarks, with pions and without quarks and the model with pions and quarks (the reference model) as OO, OQ, PO and PQ, respectively. The bag constant is $B = 250$ MeV fm$^{-3}$ for the models with quarks. In Figure 3.7, we show the mass-radius trajectories for our EOS’s together with the recent data of compact star masses. We can see that the pion population and the hadron-quark transition lower the maximum mass because they soften the EOS. For
CHAPTER 3. EQUATION OF STATE

Figure 3.7: Mass-radius trajectories for our EOS’s of NS matter. In the left panel, long-dashed, short-dashed, solid and dot-dashed lines represent the models OO (the original Shen EOS in §3.1), OQ, PO and PQ (the reference model), respectively, where the definition of the models is given in the text. In the right panel, we show the results of the models with pions, and the solid line is the same as that in the left panel (PO). Other lines correspond, from bottom to top, to the models with pions and quarks with $B = 150, 200, 250, 300, 400$ MeV fm$^{-3}$. In both panels, horizontal dotted lines represent the lower limit of the maximum mass of compact stars determined by pulsars Ter 5 I and J with 95% confidence, and the plots with 1σ error bars are for the measurements of neutron star EXO 0748-676.

instance, the maximum mass of model OO is $2.2M_\odot$ while those of models PO, OQ and PQ are $2.0M_\odot$, $1.8M_\odot$ and $1.8M_\odot$, respectively. In the right panel, we show the dependence on $B$. We can see that the maximum mass becomes lower when the bag constant becomes small. This is because the phase transition density is lower for smaller bag constants.

The recent measurements of compact star masses increase the lower limit of the maximum mass. For instance, it has been established with 95% confidence that at least one of the two pulsars, Ter 5 I and J, is more massive than $1.68M_\odot$ from analysis of the joint probabilities (Ransom et al. 2005). For the neutron star EXO 0748-676, it has been reported that the lower limits on the mass and radius are $M \geq 2.10 \pm 0.28M_\odot$ and $R \geq 13.8 \pm 1.8$ km with 1σ error bars, while there are uncertainties in the analysis of the X-ray burst spectra (Özel et al. 2006). We note that the reference model ($B = 250$ MeV fm$^{-3}$) is
consistent with these measurements whereas models with lower bag constants $B \lesssim 200 \text{ MeV fm}^{-3}$ in our study produce lower maximum masses.
Chapter 4

Systematic Study of Stellar Collapse

In this and next chapter, we investigate the iron-core collapses systematically for the mass. To be more specific, we assume that the mass of an iron core is mainly determined by the entropy per baryon, and our investigation is done systematically for entropy. The relation of mass and entropy is described in §4.1.1. Some models dealt here correspond Pop III massive stars and results of them are mainly given in the next chapter. It is also noted that, in this investigation, we utilize the EOS by Shen et al. (1998a, 1998b) described in §3.1.

4.1 Initial Models

4.1.1 Setups

Here, we prepare isentropic iron cores in unstable equilibrium as initial models of the collapse. In order to study the black hole formation, we intend to investigate higher entropy models, while the entropy per baryon for ordinary supernovae is \( s \sim 1k_B \), where \( k_B \) is the Boltzmann constant. We set the initial central temperature to be \( 7.7 \times 10^9 \) K, a typical value for the nuclear statistical equilibrium (NSE). We further assume that the initial electron fraction \( Y_e \) equals to 0.5 in the whole core. Hence the entropy per baryon in the iron core is the only parameter which characterizes 24 models investigated here (Table 4.1). In order to examine the ambiguity in the onset of collapse, we also adopt model E2v which has the same initial entropy per baryon as model E2 (\( s = 4k_B \)) but has half the central density.

We solve the Oppenheimer-Volkoff equation to obtain the equilibrium
CHAPTER 4. SYSTEMATIC STUDY OF STELLAR COLLAPSE

Table 4.1: Initial Model Entropy, Mass and Radius

<table>
<thead>
<tr>
<th>model</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
<th>E5</th>
<th>E6</th>
<th>F1</th>
<th>F2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$ ($k_B$)</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>7.5</td>
<td>10.0</td>
<td>13.0</td>
<td>15.98</td>
<td>17.50</td>
</tr>
<tr>
<td>$M_{\text{iron}}$ ($M_\odot$)</td>
<td>2.44</td>
<td>3.49</td>
<td>4.97</td>
<td>10.6</td>
<td>19.3</td>
<td>34.0</td>
<td>53.4</td>
<td>65.0</td>
</tr>
<tr>
<td>$R_{\text{iron}}$ (km)</td>
<td>267</td>
<td>370</td>
<td>475</td>
<td>743</td>
<td>1030</td>
<td>1380</td>
<td>1730</td>
<td>1910</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>model</th>
<th>F3</th>
<th>F4</th>
<th>F5</th>
<th>F6</th>
<th>F7</th>
<th>F8</th>
<th>F9</th>
<th>F10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$ ($k_B$)</td>
<td>19.17</td>
<td>20.96</td>
<td>22.97</td>
<td>25.25</td>
<td>27.77</td>
<td>30.54</td>
<td>33.74</td>
<td>37.19</td>
</tr>
<tr>
<td>$M_{\text{iron}}$ ($M_\odot$)</td>
<td>79.3</td>
<td>96.3</td>
<td>117</td>
<td>144</td>
<td>176</td>
<td>215</td>
<td>265</td>
<td>324</td>
</tr>
<tr>
<td>$R_{\text{iron}}$ (km)</td>
<td>2120</td>
<td>2330</td>
<td>2570</td>
<td>2840</td>
<td>3150</td>
<td>3480</td>
<td>3860</td>
<td>4280</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>model</th>
<th>F11</th>
<th>F12</th>
<th>F13</th>
<th>F14</th>
<th>F15</th>
<th>F16</th>
<th>F17</th>
<th>F18</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$ ($k_B$)</td>
<td>40.96</td>
<td>45.26</td>
<td>49.89</td>
<td>55.50</td>
<td>61.16</td>
<td>67.78</td>
<td>74.75</td>
<td>84.06</td>
</tr>
<tr>
<td>$M_{\text{iron}}$ ($M_\odot$)</td>
<td>396</td>
<td>486</td>
<td>594</td>
<td>738</td>
<td>904</td>
<td>1100</td>
<td>1340</td>
<td>1690</td>
</tr>
<tr>
<td>$R_{\text{iron}}$ (km)</td>
<td>4730</td>
<td>5250</td>
<td>5810</td>
<td>6480</td>
<td>7160</td>
<td>7940</td>
<td>8770</td>
<td>9860</td>
</tr>
</tbody>
</table>

$s$, $M_{\text{iron}}$ and $R_{\text{iron}}$ denote the entropy per baryon in the iron core, the mass of iron core and the radius of the iron core, respectively.

configurations. Oppenheimer-Volkoff equation is described as,

$$\frac{dp(r)}{dr} = -\frac{G(M(r)+\frac{4\pi r^3 p(r)}{c^2})}{r^2} \rho(r) + \frac{\rho(r)}{c^2} \frac{1}{1-\frac{2GM(r)}{c^2r}}. \quad (4.1)$$

where $\rho$ is the energy density, $p$ is the pressure, $M$ is the enclosed gravitational mass, $r$ is the circumference radius, $G$ is the gravitational constant and $c$ is the velocity of light. We define the mass of iron core, $M_{\text{iron}}$, as the mass coordinate where the temperature is $5 \times 10^9$ K, whereas we set the outer boundary at a much larger radius so as not to affect the dynamics.

4.1.2 Natural Counterparts of the Models

Here, we discuss the natural counterparts of our models. For the models with the initial entropy $s \geq 15.98k_B$ (models F1-F18), we can associate them to Pop III massive stars as below. Pop III massive stars with the mass $\gtrsim 260M_\odot$ form an iron core at the center during the collapse by the pair-instability. This iron core is isentropic and the entropy per baryon is determined by the oxygen core mass at the onset of collapse (Bond et al. 1984). On the other hand, Bond et al. (1984) also estimated the relation between the initial stellar mass and the helium core mass for Pop III massive stars. Although the fraction of helium burnt to oxygen in the helium core is uncertain, recent
Table 4.2: Correspondence of the Initial Mass to the Iron Core Entropy for Population III Star Models

<table>
<thead>
<tr>
<th>Model</th>
<th>$M_i$ ($M_\odot$)</th>
<th>$M_{\text{He}}$ ($M_\odot$)</th>
<th>$M_{\text{O}}$ ($M_\odot$)</th>
<th>$s_{\text{O}}$ ($k_B$)</th>
<th>$s_{\text{iron}}$ ($k_B$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>300</td>
<td>159</td>
<td>143</td>
<td>14.54</td>
<td>15.98</td>
</tr>
<tr>
<td>F2</td>
<td>375</td>
<td>201</td>
<td>181</td>
<td>16.06</td>
<td>17.50</td>
</tr>
<tr>
<td>F3</td>
<td>470</td>
<td>254</td>
<td>228</td>
<td>17.73</td>
<td>19.17</td>
</tr>
<tr>
<td>F4</td>
<td>585</td>
<td>319</td>
<td>287</td>
<td>19.53</td>
<td>20.96</td>
</tr>
<tr>
<td>F5</td>
<td>730</td>
<td>400</td>
<td>360</td>
<td>21.54</td>
<td>22.97</td>
</tr>
<tr>
<td>F6</td>
<td>915</td>
<td>504</td>
<td>454</td>
<td>23.81</td>
<td>25.25</td>
</tr>
<tr>
<td>F7</td>
<td>1145</td>
<td>633</td>
<td>570</td>
<td>26.33</td>
<td>27.77</td>
</tr>
<tr>
<td>F8</td>
<td>1430</td>
<td>794</td>
<td>714</td>
<td>29.10</td>
<td>30.54</td>
</tr>
<tr>
<td>F9</td>
<td>1800</td>
<td>1001</td>
<td>901</td>
<td>32.29</td>
<td>33.74</td>
</tr>
<tr>
<td>F10</td>
<td>2250</td>
<td>1254</td>
<td>1129</td>
<td>35.74</td>
<td>37.19</td>
</tr>
<tr>
<td>F11</td>
<td>2800</td>
<td>1563</td>
<td>1407</td>
<td>39.51</td>
<td>40.96</td>
</tr>
<tr>
<td>F12</td>
<td>3500</td>
<td>1956</td>
<td>1760</td>
<td>43.79</td>
<td>45.26</td>
</tr>
<tr>
<td>F13</td>
<td>4350</td>
<td>2434</td>
<td>2191</td>
<td>48.44</td>
<td>49.89</td>
</tr>
<tr>
<td>F14</td>
<td>5500</td>
<td>3080</td>
<td>2772</td>
<td>54.04</td>
<td>55.50</td>
</tr>
<tr>
<td>F15</td>
<td>6800</td>
<td>3810</td>
<td>3429</td>
<td>59.70</td>
<td>61.16</td>
</tr>
<tr>
<td>F16</td>
<td>8500</td>
<td>4765</td>
<td>4288</td>
<td>66.32</td>
<td>67.78</td>
</tr>
<tr>
<td>F17</td>
<td>10500</td>
<td>5889</td>
<td>5300</td>
<td>73.29</td>
<td>74.75</td>
</tr>
<tr>
<td>F18</td>
<td>13500</td>
<td>7574</td>
<td>6817</td>
<td>82.59</td>
<td>84.06</td>
</tr>
</tbody>
</table>

$M_i$, $M_{\text{He}}$ and $M_{\text{O}}$ denote the initial stellar mass, helium core mass and oxygen core mass, respectively. The entropy per baryon in the oxygen core and the iron core is denoted as $s_{\text{O}}$ and $s_{\text{iron}}$, respectively.

Theoretical calculations suggest that most of helium is consumed (Umeda & Nomoto 2002). Therefore, assuming that 90% of helium becomes oxygen, we obtain the correspondence of the initial mass to the entropy per baryon of iron core as shown in Table 4.2. We find that the mass of oxygen core and the entropy of iron core estimated this way close to those obtained recently by Ohkubo et al. (2006) for the realistic progenitor model. For example, they found $M_{\text{O}} \sim 240 M_\odot$ and $s_{\text{iron}} \sim 19.6k_B$ for their models of Pop III star with $500M_\odot$. In addition, the stars with an oxygen core whose mass is $\gtrsim 8000 M_\odot$ become unstable due to general relativistic effects before they encounter the pair-instability (Bond et al. 1984). In our analysis, the oxygen core with $8000 M_\odot$ corresponds to the initial mass $\sim 16,000 M_\odot$, the upper limit of the pair-instability collapse given above.
According to the evolutionary calculation by Nomoto et al. (2005), the iron core of the Pop III star with $100M_\odot$ has an entropy of $\sim 3.5k_B$. Thus we may be able to associate our models with initial entropy $s \lesssim 4k_B$, namely models E1 and E2, with Pop III massive stars of $\lesssim 100M_\odot$. These stars do not collapse by the pair-instability and form the iron core as ordinary Pop I stars. The core made in such a star is not isentropic exactly, but the value of entropy is close to our models. Moreover, our models may correspond to stars made by the stellar collision in the cluster as below. Suzuki et al. (2007) studied the structures and evolution of these merged stars in the hydrogen-burning stage. According to them, the smaller star sits at the center of the larger star after the merger of two stars with different masses. It is also demonstrated that the merged stars become convectively unstable by the positive gradient of the mean molecular weight and that their evolution thereafter approach those of the single homogeneous star with the same mass and abundance. The central entropies of the merged stars will then be larger than those of the inhomogeneous single stars with the same mass. According to the $N$-body simulations, merged stars grow to $\gtrsim 100M_\odot$. Since the single star with $100M_\odot$ has an entropy of $\sim 3.5k_B$, entropy of these stars may be $\gtrsim 3k_B$.

When the pair instability occurs, there is a possibility for merged stars to explode as a pair instability supernova. According to the discussion given above, the iron cores with $\lesssim 15k_B$ will not be formed. However, even if the pair instability occurs, the iron cores in this regime may still be formed. In fact, the positive entropy gradient and/or rotation may suppress the convection in the merged star, and the entropy at the center may remain low after the merger. Then the merged star has a massive envelope with a smaller core than the single stars with the same total mass. If the pair instability occurs for these objects, the nuclear burning may not produce total disruptions but lead to the eventual collapse. Again inferring from single Pop III stars, we speculate that the central entropies of the merged stars may be smaller than $s \sim 15k_B$, which is smallest value not to make pair-instability supernovae for the single Pop III stars. It is incidentally mentioned that the relations between the total mass and the iron-core mass of merged black-hole progenitors is highly uncertain at present.

### 4.2 Results

In this section, we show the results of our computations and discuss them. To overview the characteristics of the models which we surveyed, we show in Figure 4.1, the evolution of the central density and temperature of our results...
CHAPTER 4. SYSTEMATIC STUDY OF STELLAR COLLAPSE

Figure 4.1: The evolution of the central density and temperature of our results except for red line, which is same as that of Figure 1.2. Orange, light green, green, light blue and blue line are for E1 ($s = 3k_B$), E3 ($s = 5k_B$), E4 ($s = 7.5k_B$), F2 ($s = 17.5k_B$) and F17 ($s = 74.75k_B$) model.

together with that of another simulation of the supernovae explosion. From this figure, we can recognize that our investigation covers very broad region in the plane and the current models are sufficient for systematic studies.

4.2.1 Dynamical Features

It is known that ordinary supernovae with $s \sim 1k_B$ bounce because their central density exceeds the nuclear density ($\sim 2.5 \times 10^{14}\text{g cm}^{-3}$) and pressure drastically increases. From our computations, we find that models with $3k_B \leq s \leq 7.5k_B$ ($M_{\text{iron}} \leq 10.6M_\odot$) have a bounce and that they recollapse to black holes. On the other hand, models with $s > 7.5k_B$ ($M_{\text{iron}} > 10.6M_\odot$) collapse to black holes directly without bounce. We show the evolution of core collapse in Figure 4.2 for two representative cases.

In the case of $3k_B \leq s \leq 7.5k_B$, it is noted that the bounce mecha-
nism of the core is not the same as that of ordinary supernovae. The high entropy cores bounce because of the thermal pressure of nucleons at sub-nuclear density. We can see this fact from the evolutions of central density and temperature in the phase diagram of the nuclear matter at $Y_e = 0.4$ and $0.2$ (Figure 4.3). We note that for all models at the center, $Y_e \sim 0.4$ and $Y_e \sim 0.2$ when $T \sim 1 \text{ MeV}$ and $T \sim 10 \text{ MeV}$, respectively. These figures show that the models with higher entropies go from the non-uniform mixed phase of nuclei and free nucleons to the classical ideal gas phase of thermal nucleons and $\alpha$ particles, whereas that of an ordinary supernova goes into the uniform nuclear matter phase. In the ideal gas phase, the number of non-relativistic nucleons and $\alpha$ particles is comparable to that of relativistic electrons. Since the adiabatic index of non-relativistic gas is $\gamma = \frac{5}{3}$ and that of relativistic gas is $\gamma = \frac{4}{3}$, the collapse is halted and bounce occurs.

Because this bounce is weak and the shock is stalled, the inner core (or the protoneutron star) grows beyond the maximum mass of the neutron star and recollapses to a black hole soon (left panel of Figure 4.2). In Figure 4.4 we show the maximum mass of the neutron star assuming isentropy and the constant electron fraction ($Y_e = 0.1$) under the EOS by Shen et al. (1998a, 1998b) described in § 3.1. It is noted that the maximum mass is larger than $3M_\odot$ for the neutron star with high entropies, $s \gtrsim 4k_B$. Since the maximum
Figure 4.3: Phase diagram in $\rho - T$ plane from Shen et al. (1998b) for fixed electron fraction, $Y_e$ (thick lines). The nucleus exists in the region below these thick lines. The phase boundaries depend on $Y_e$, whereas the same trajectories are plotted for the upper panel and the lower panel. The dashed line represents the evolution of the central density and temperature for the ordinary supernova progenitor with the initial mass $15M_\odot$ (Sumiyoshi et al. 2005), and the solid lines do the same for the progenitors studied. Each line corresponds to models E1 ($s = 3k_B$), E3 ($s = 5k_B$) and E5 ($s = 10k_B$), from right to left.

mass of the neutron star depends on the EOS, it should be remind that the time interval from the bounce to the recollapse also depends on it (Sumiyoshi et al. 2006).

In Table 4.3 we show the inner core mass, central density, temperature and adiabatic index at the bounce together with the interval time from the bounce to the apparent horizon formation. We can recognize that the density and the adiabatic index at the bounce get lower for the models with higher initial entropies. These features indicate that the bounce is not due to the nuclear force but to the thermal pressure of non-relativistic gas for high entropy cores. Moreover, the interval time from the bounce to the apparent horizon formation is shorter for the higher entropy cores. This is because the initial mass of the iron core ($M_{\text{iron}}$) is larger than the maximum mass of the neutron star ($M_{\text{max}}$) for the models with high entropies and they can collapse to black holes quickly.

We compare our results with other studies. In Sekiguchi & Shibata (2005),
Figure 4.4: The maximum mass of the neutron star assuming isentropy and a constant electron fraction which is isentropic ($Y_e = 0.1$) under the EOS by Shen et al. (1998a, 1998b). The solid and dashed lines represent the maximum mass in the sense of the baryon rest mass and the gravitational mass, respectively.

The non-rotating models with an iron core mass of $\gtrsim 2.28M_\odot$ end up with black holes without bounce. In our models, on the other hand, it is shown that the iron core with $\lesssim 10.6M_\odot$ (or the initial entropy $s \lesssim 7.5k_B$) has bounce before black hole formation. This discrepancy comes from the fact that their EOS is parametric and does not take into account properly the effects of thermal nucleons in the collapsing phase. On the other hand, the rotating Population III star with $\sim 300M_\odot$ has a weak bounce at $\rho_c \sim 10^{12}\text{g cm}^{-3}$ in Fryer et al. (2001), and these authors adopt a realistic EOS (Herant et al. 1994). Since the rotation tends to produce a bounce, we can predict that the bounce is inevitable for an iron core with the mass $\lesssim 10M_\odot$ irrespective of rotations, and the effects of thermal nucleons are crucial.

In the high entropy case with $s > 7.5k_B$, more massive cores do not have a bounce but form an accretion shock before the apparent horizon formation. This is because, the outer region keeps collapsing supersonically while the central region becomes gravitationally stable by the thermal pressure of non-relativistic gas. We can see this feature in the right panel of Figure 4.2. As the initial mass gets larger, the transition occurs smoothly from the collapse with bounce to the one without bounce. Incidentally, all models which correspond to the Pop III massive stars do not have a bounce and their features are stated
### Table 4.3: Key Parameters for Models with Bounce.

<table>
<thead>
<tr>
<th>Model</th>
<th>( s_{\text{initial}} ) ((k_B))</th>
<th>( M_{\text{iron}} ) ((M_\odot))</th>
<th>( \rho_{\text{initial}} ) ((g \text{ cm}^{-3}))</th>
<th>( T_{\text{initial}} ) ((K))</th>
<th>( M_{\text{bounce}} ) ((M_\odot))</th>
<th>( \rho_{\text{bounce}} ) ((g \text{ cm}^{-3}))</th>
<th>( T_{\text{bounce}} ) ((\text{MeV}))</th>
<th>( \gamma_{\text{bounce}} )</th>
<th>( t_{\text{recollapse}} ) ((\text{msec}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>3.0</td>
<td>2.44</td>
<td>2.71 \times 10^8</td>
<td>7.75 \times 10^7</td>
<td>0.75</td>
<td>1.95 \times 10^{14}</td>
<td>25.9</td>
<td>2.38</td>
<td>96.7</td>
</tr>
<tr>
<td>E2</td>
<td>4.0</td>
<td>3.49</td>
<td>1.40 \times 10^8</td>
<td>7.75 \times 10^9</td>
<td>1.10</td>
<td>9.58 \times 10^{13}</td>
<td>26.9</td>
<td>1.89</td>
<td>62.0</td>
</tr>
<tr>
<td>E2v</td>
<td>4.0</td>
<td>2.93</td>
<td>7.00 \times 10^7</td>
<td>6.86 \times 10^8</td>
<td>1.05</td>
<td>9.90 \times 10^{13}</td>
<td>26.7</td>
<td>1.91</td>
<td>63.4</td>
</tr>
<tr>
<td>E3</td>
<td>5.0</td>
<td>4.97</td>
<td>8.82 \times 10^7</td>
<td>7.75 \times 10^9</td>
<td>1.5</td>
<td>2.97 \times 10^{13}</td>
<td>19.4</td>
<td>1.58</td>
<td>52.6</td>
</tr>
<tr>
<td>E4</td>
<td>7.5</td>
<td>10.6</td>
<td>4.20 \times 10^7</td>
<td>7.75 \times 10^9</td>
<td>2.7</td>
<td>3.00 \times 10^{12}</td>
<td>12.0</td>
<td>1.54</td>
<td>37.9</td>
</tr>
</tbody>
</table>

*\( s_{\text{initial}} \) is the initial value of the entropy per baryon. \( M_{\text{iron}} \) and \( M_{\text{bounce}} \) are the mass of initial iron core and inner core at the bounce \((t = 0)\), respectively. \( \rho_{\text{initial}} \) and \( \rho_{\text{bounce}} \) are the central density of the initial model and at the bounce, respectively. \( T_{\text{initial}} \) and \( T_{\text{bounce}} \) are the central temperature of the initial model and at the bounce, respectively. \( \gamma_{\text{bounce}} \) is the central adiabatic index at the bounce. \( t_{\text{recollapse}} \) is the interval time from the bounce to the apparent horizon formation.*

in the next chapter.

### 4.2.2 Neutrino Signals

In this section, we discuss neutrino emission during core collapse. As mentioned already, we compute the collapse until after the apparent horizon is formed inside the inner core. However, the location of the event horizon is not known for our models while it is proved mathematically that the event horizon is always located outside the apparent horizon. Although it is likely, we do not know if the event horizon is also inside the inner core, since the null geodesics should be integrated to infinity, which is impossible. If this is the case, the black hole formation is not reflected in the neutrino signal until the inner core is swallowed by the event horizon because the inner core is the main source of neutrinos. Unfortunately, the numerical difficulty does not allow us to compute the dynamics until the apparent horizon swallows the inner core entirely, which is the sufficient condition that is the event horizon is outside the inner core. From the extrapolation of the obtained evolutions, we expect that it will take the apparent horizon \( \sim 1 \) ms to reach the inner core surface for all models. The neutrino emission from the inner core is estimated to be negligible during this period. Hence, we have to somehow estimate the neutrinos diffusing out of the region between the inner core surface and the neutrino sphere. We can obtain the upper and lower limits as follows. The upper limit is estimated assuming that all neutrinos in this region flow out without absorbed or scattered. As for the lower limit, on the other hand, we assume that all neutrinos in this region are trapped and do
and that the luminosities of $\nu$ depends on the EOS. In Sumiyoshi et al. (2006), it is shown that the features mentioned, the time interval from the bounce to the apparent horizon formation sphere flow freely after the apparent horizon formation. As already men-

eral models under the assumption that the neutrinos outside the neutrino core. This is because the duration of the neutrino emission is longer for the lower entropy models, while the duration is shorter and the neutrino emission is not so small. In the following, ignoring this tiny difference, we denote these four species as $\nu_x$ collectively. In Table 4.4, we can recognize that the total energy does not change monotonically with the initial entropy of the core. This is because the duration of the neutrino emission is longer for the lower entropy models, while the duration is shorter and the neutrino luminosity is larger for the higher entropy models.

In Figure 4.5, we show the time evolutions of neutrino luminosity for several models under the assumption that the neutrinos outside the neutrino sphere flow freely after the apparent horizon formation. As already mentioned, the time interval from the bounce to the apparent horizon formation depends on the EOS. In Sumiyoshi et al. (2006), it is shown that the features
Figure 4.5: Luminosities of $\nu_e$ (short-dashed line), $\bar{\nu}_e$ (solid line) and $\nu_x$ (long-dashed line) as a function of $t$, where $\nu_x$ stands for $\mu$- and $\tau$-neutrinos and their anti-particles. Squares show the time when the apparent horizon is formed. Upper left, upper right, lower left and lower right panels are for models E1 ($s = 3k_B$), E2 ($s = 4k_B$), E3 ($s = 5k_B$) and E4 ($s = 7.5k_B$), respectively.
Figure 4.6: Snapshots of the profiles for the luminosity and the chemical potential of an electron-type neutrino. The left panel corresponds to the model E1 ($s = 3k_B$) and the right to the model E4 ($s = 7.5k_B$).

of the neutrino emission, such as a neutronization burst, are not sensitive to the EOS very much for the early phase. From Figure 4.5 we can see that the sign of neutronization burst becomes less remarkable and disappears for the higher entropy models.

In order to analyze these features, we discuss the neutrino emission from model E1, as a reference model. In the upper left panel of Figure 4.6, we show snapshots of the luminosity of an electron-type neutrino as a function of the baryon mass coordinate. We can recognize that neutrinos are emitted on the shock surface mainly. The luminosity on the shock surface has a peak (e.g., at $1.25M_\odot$ in the upper left panel of Figure 4.6), which is similar to the situation for ordinary supernovae (e.g., Thompson et al. 2003). In the following, we estimate the value of the luminosity semi-analytically and compare it with the results of our numerical simulations.

At first, the number density of neutrinos on the shock surface can be evaluated roughly by the equilibrium value,

$$n_{eq}(\epsilon)d\epsilon \propto \frac{\epsilon^2}{\exp \left( \frac{\epsilon - \mu_\nu}{k_BT} \right) + 1}d\epsilon,$$

where $T$ and $\mu_\nu$ are the temperature and the chemical potential of the electron-type neutrino in $\beta$-equilibrium at the shock surface, respectively.
Figure 4.7: Time evolutions of the number flux for the electron-type neutrino with the energy $10 \text{ MeV} < E < 20 \text{ MeV}$ detected by the comoving observer. Solid lines and dashed lines represent the results of our computation and the values estimated by the number density in equilibrium and $\langle \cos \theta \rangle$ of our computation, respectively. The left panel corresponds to model E1 ($s = 3k_B$) at $M = 1.3M_\odot$ and the right to model E4 ($s = 7.5k_B$) at $M = 3.3M_\odot$.

Here the value $\mu_\nu$ is defined as $\mu_\nu \equiv \mu_e - (\mu_n - \mu_p)$, where $\mu_e$, $\mu_n$ and $\mu_p$ are the chemical potentials of electron, neutron and proton, respectively, and they are given in the EOS by Shen et al. (1998a, 1998b). The number flux is estimated as $c n_{eq} \langle \cos \theta \rangle$, where $\langle \cos \theta \rangle$ is a mean value of the angular cosine over the neutrino angular distribution and $c$ is the light velocity. In Figure 4.7 we compare the results of our numerical computation with the number flux estimated above. We can see that the equilibrium is not achieved completely, but the fraction is rather constant, $\sim 0.6$. Therefore, the luminosity is well estimated by

$$L(\epsilon) d\epsilon = C \frac{16\pi^2 r^2 \langle \cos \theta \rangle \epsilon^3}{h^3 c^2 \left( \exp \left( \frac{\epsilon - \mu_\nu}{k_B T} \right) + 1 \right)} d\epsilon,$$

where $h$ is the Planck constant, $r$ is the radius of the shock surface and $C \sim 0.6$.

From equation (4.3), we can see that the luminosity is determined by $r$, $\mu_\nu$, $T$ and $\langle \cos \theta \rangle$. According to our numerical computation, $T \sim 1.5 \text{ MeV}$ and $\langle \cos \theta \rangle \sim 0.5$ do not change very much on the time scale of the neutronization burst. Thus, the luminosity is dictated mainly by $r$ and $\mu_\nu$. Snapshots
CHAPTER 4. SYSTEMATIC STUDY OF STELLAR COLLAPSE

Figure 4.8: Spectra of time-integrated emissions of $\nu_e$ (short-dashed line), $\bar{\nu}_e$ (solid line) and $\nu_x$ (long-dashed line). Upper left, upper right, lower left and lower right panels are for models E2 ($s = 4k_B$), E4 ($s = 7.5k_B$), E5 ($s = 10k_B$) and E6 ($s = 13k_B$), respectively.


Of the profiles of $\mu_\nu$ are shown in the lower left panel of Figure 4.6, and we can see that $\mu_\nu$ has a peak on the shock surface for the following reason. When matter accretes onto the shock, the baryon mass density and the electron number density rise, leading to the increase of $\mu_e$ and, as a result, $\mu_\nu$. Immediately thereafter, neutronization occurs and the value of $(\mu_n - \mu_p)$ rises, which reduces $\mu_\nu$. We can recognize from Figure 4.6 that the peaks of $\mu_\nu$ and the luminosity are correlated. As for the time evolution, the luminosity on the shock surface is lower at the early phase because the shock radius is small. On the other hand, it is also lower at the late phase because $\mu_\nu$ is lower. This is the reason why the luminosity on the shock surface has a peak.

We now investigate model E4, whose initial entropy is $s = 7.5k_B$. In the lower right panel of Figure 4.6, snapshots of the profiles of $\mu_\nu$ for model E4 are shown. We can see that the value of $\mu_\nu$ at the shock surface is lower than that of model E1 at the early phase. This is because the baryon mass density on the shock surface of model E4 at the bounce is lower than that of model E1, as has been mentioned. Accordingly, the electron number density and $\mu_e$ are also lower for model E4, and $\mu_\nu$ does not rise so high. This is the main reason why the neutronization burst is not remarkable. It is noted, moreover, that the electron fraction, $Y_e$, on the shock surface of model E4
Figure 4.9: Spectra of time-integrated emissions of $\nu_e$ (short-dashed line), $\bar{\nu}_e$ (solid line) and $\nu_x$ (long-dashed line). The upper left and upper right panels give the time integrations of the emission before and after bounce, respectively, for model E2 ($s = 4k_B$). The lower left and lower right panels present the emission before and after shock formation, respectively, for model E6 ($s = 13k_B$).

is lower than that of model E1. This is because nuclei do not exist and the nucleons are already neutronized on the shock surface. The absence of nuclei is consistent with the fact that the higher the initial entropy is, the earlier nuclei dissolve into nucleons, as explained by Figure 4.3. In addition, we can see that the luminosity of $\nu_e$ rises monotonically. This is because the area of a shock surface increases whereas $\mu_\nu$ is almost unchanging.

We show the time-integrated neutrino spectra in Figure 4.8. We can see that the spectra become softer for higher entropy models, especially for $\bar{\nu}_e$ and $\nu_x$. In order to investigate this tendency, we show the time-integrated spectra of the neutrino emitted before and after the shock formation in Figure 4.9. We can see that, for higher entropy models, $\bar{\nu}_e$ and $\nu_x$ are also emitted before the shock formation. They are created by the electron-positron pair annihilation, and their energy is relatively lower ($\lesssim$ several MeV) because the temperature is low ($T \lesssim 1$ MeV). On the other hand, for lower entropy models, $\bar{\nu}_e$ and $\nu_x$ can not be produced by the electron-positron pair process because positrons are absent owing to Pauli blocking. As for the $\bar{\nu}_e$ and $\nu_x$ emitted after the shock formation, they are mainly created by bremsstrahlung. In this phase, the temperature near the neutrino sphere
Figure 4.10: Comparisons of the density profiles (left) and the velocity profiles (right). Solid lines represent the initial profiles for model E2; dashed lines represent the profiles for model E2v at the time when the central density becomes the same as the initial central density of model E2.

rises to $T \sim$ several MeV, which makes the neutrino energies relatively high, $\sim 10$ MeV. Since the low energy ($\lesssim$ several MeV) neutrinos are not emitted to any great extent and the spectra become harder for lower entropy models, the emission of low energy $\bar{\nu}_e$ and $\nu_x$ is characteristic for the collapse of high entropy cores.

### 4.2.3 Initial Velocity Dependence

We compare the results of models E2 and E2v, which are different in the initial values of central density and temperature, but have the same initial values of entropy per baryon ($s = 4k_B$). We can consider that models E2 and E2v are the same model but with different initial velocities, because the density profile of model E2v at the time when the central density reaches that of the initial model of E2 almost coincides with that of model E2 (Figure 4.10). In reality, the onset of a collapse is determined not only by the core structure but also by the whole stellar structure. Thus, studying the initial velocity dependence of the core is meaningful.

As a result of this comparison, we find that the initial velocity does not affect crucially the ensuing dynamics and the features of emitted neutrinos such as total number spectra or the time evolutions of the luminosity. This is
because the velocity of model E2v at the time in Figure 4.10 is several times lower than the sound speed at each point. For instance, the fastest point of model E2v in Figure 4.10 has the velocity \( \sim 10^8 \text{cm s}^{-1} \) while the sound speed is \( \sim 7 \times 10^8 \text{cm s}^{-1} \), there. If the supersonic region, where the infalling velocity exceeds the sound speed, existed in the initial model, the initial velocity profile may be important for the dynamics. However, since the temperature of our initial models is slightly higher than the critical temperature for the photodisintegration instability, they are unlikely to have supersonic region.

\[ \text{CHAPTER 4. SYSTEMATIC STUDY OF STELLAR COLLAPSE} \]

\[ \text{55} \]

\[ \because \text{the velocity of model E2v at the time in Figure 4.10 is several times lower than the sound speed at each point. For instance, the fastest point of model E2v in Figure 4.10 has the velocity } \sim 10^8 \text{cm s}^{-1} \text{ while the sound speed is } \sim 7 \times 10^8 \text{cm s}^{-1} \text{, there. If the supersonic region, where the infalling velocity exceeds the sound speed, existed in the initial model, the initial velocity profile may be important for the dynamics. However, since the temperature of our initial models is slightly higher than the critical temperature for the photodisintegration instability, they are unlikely to have supersonic region.} \]

\[ \text{4.3 Astrophysical Implications} \]

In this section, we assume that models E1-E6 correspond to the merged stars which collapse to IMBHs at the center of our Galaxy (\( \sim 8.5 \text{kpc from the sun} \)). We estimate the neutrino event number for some detectors. In the following estimations for the neutrino event number, we do not take into account the neutrino mixing, although it should be. Since the mixing occurs mainly in the resonance regions and they are located outside the iron core of the progenitor, the neutrino oscillation does not affect the dynamics of core. Unfortunately the structures of the envelopes of merged stars, which are crucial for the neutrino mixing, are quite uncertain. There remain uncertainties as well on the mixing parameters, such as the mixing angle of \( \sin^2 2\theta_{13} \) or the mass hierarchy. Thus, the precise evaluation of the neutrino flux including the neutrino mixing is deferred to future study, however, we introduce the method to calculate the neutrino mixing in Appendix B.

\[ \text{4.3.1 Detection of Low Energy } \bar{\nu}_e \text{ by SK III and KamLAND} \]

As already mentioned, a good deal of low energy \( \bar{\nu}_e \) is emitted from the collapse of the high entropy cores, which softens the spectrum. We estimate the \( \bar{\nu}_e \) event number for SK III and KamLAND, currently operating neutrino detectors, under the assumption that the black hole formations considered in former sections occur at the center of our Galaxy. For both detectors, the dominant reaction is the inverse beta decay,

\[ \bar{\nu}_e + p \longrightarrow e^+ + n, \quad (4.4) \]

which we take into account only. We adopt the cross section for this reaction from Vogel & Beacom (1999). For SK III, we assume that the fiducial volume is 22.5 kton and the trigger efficiency is 100\% at 4.5 MeV and 0\% at 2.9 MeV, which are the values at the end of Super Kamiokande I (Hosaka et al. 2006).
Table 4.5: Event Numbers of $\bar{\nu}_e$ from SK III and KamLAND.

<table>
<thead>
<tr>
<th>model</th>
<th>$\frac{N_{\bar{\nu}<em>e &lt; 10 \text{ MeV}}}{N</em>{\bar{\nu}_e \text{ SK}}}$</th>
<th>$\frac{N_{\bar{\nu}<em>e \text{ SK}}}{N</em>{\bar{\nu}_e \text{ Kam}}}$</th>
<th>$\frac{N_{\bar{\nu}<em>e &lt; 10 \text{ MeV}}}{N</em>{\bar{\nu}_e \text{ Kam}}}$</th>
<th>$N_{\bar{\nu}_e \text{ Kam}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>3.3%</td>
<td>6163</td>
<td>3.3%</td>
<td>174</td>
</tr>
<tr>
<td>E2</td>
<td>4.0%</td>
<td>4778</td>
<td>4.0%</td>
<td>135</td>
</tr>
<tr>
<td>E3</td>
<td>4.6%</td>
<td>4319</td>
<td>4.6%</td>
<td>122</td>
</tr>
<tr>
<td>E4</td>
<td>7.3%</td>
<td>4018</td>
<td>7.3%</td>
<td>114</td>
</tr>
<tr>
<td>E5</td>
<td>11.8%</td>
<td>5326</td>
<td>12.0%</td>
<td>151</td>
</tr>
<tr>
<td>E6</td>
<td>20.1%</td>
<td>9139</td>
<td>20.5%</td>
<td>259</td>
</tr>
</tbody>
</table>

The subscript “$< 10 \text{ MeV}$” means the event of $\bar{\nu}_e$ with $< 10 \text{ MeV}$, and the subscript “SK” and “Kam” mean the prediction for SK III and KamLAND, respectively.

For KamLAND, we assume 1 kton fiducial mass, which means that $8.48 \times 10^{31}$ free protons are contained (Eguchi et al. 2003). We also assume that the trigger efficiency is 100% for all $\bar{\nu}_e$ energy larger than the threshold energy of the reaction.

The results are given in Table 4.5. The total event number does not change monotonically with the initial entropy of the core because the total number of neutrinos depends on both the core mass and the duration time of neutrino emission, as already mentioned. In order to investigate the hardness of $\bar{\nu}_e$ spectrum, we calculate the ratio of the event number by $\bar{\nu}_e$ with $< 10 \text{ MeV}$ to that for all events. The ambiguity about the distance of source is also canceled by this normalization. This ratio gets larger as the entropy of the core becomes higher. This suggests that we can probe the entropy of the black hole progenitor especially in higher regimes ($s \geq 7.5k_B$) because the event numbers of $\bar{\nu}_e$ with $< 10 \text{ MeV}$ are over 100 by SK III.

### 4.3.2 Detection of Neutronization Burst by SNO

The SNO detector consists of 1 kton of pure heavy water (D\(_2\)O) and can distinguish $\nu_e$ flux by the charged-current reaction of the deuterium disintegration. Since SNO can also detect the $\bar{\nu}_e$ flux, we can estimate the intensity of the neutronization burst by comparing the event from the charged-current reaction of $\nu_e$,

$$\nu_e + d \rightarrow p + p + e^-,$$  \hspace{1cm} (4.5)

and that of $\bar{\nu}_e$,

$$\bar{\nu}_e + d \rightarrow n + n + e^+,$$  \hspace{1cm} (4.6)
The values in Table 4.6 are the event number for the charged-current reaction except $N_{NC,<0.06s}$ and $N_{NC,\text{all}}$. The subscript “$<0.06s$” means the event at $t<0.06s$, where $t$ is the time measured from the bounce, and the subscript “all” means the event for all duration times of neutrino emission.

using the SNO detector. SNO can also detect the neutral-current reaction,

$$\nu + d \rightarrow n + p + \nu,$$  \hspace{1cm} (4.7)

for all species. It is noted that the neutral-current reaction contains $\nu_x (=\nu_\mu, \bar{\nu}_\mu, \nu_\tau \text{ and } \bar{\nu}_\tau)$ and the neutrino sphere of $\nu_x$ differs more from that of $\nu_e$ than that of $\bar{\nu}_e$ in general. Thus, for the comparison with reaction (4.5), reaction (4.6) is more appropriate than reaction (4.7). On the other hand, we also use (4.7) for the comparison because the event number of (4.7) is larger than that of (4.6). In our calculation, we use the cross sections from Ying et al. (1989) and assume that the trigger efficiency of these reactions is 100%. In fact, it is $\sim 92\%$ these days, which is the neutron (in the right hand side of equation (4.7)) capture efficiency on $^{35}\text{Cl}$ and deuterons (Oser 2005).

In the following analysis, we regard the emission of neutrinos before $t=0.06s$ as the neutronization burst, where the time $t$ is measured from the bounce. The criterion $t=0.06s$ is chosen empirically from our simulations as an expedient. The method for extracting the neutronization burst from detection should be reconsidered for more detailed studies. Here we calculate the event numbers for $t<0.06s$ as well as those for the entire duration time of the neutrino emission, and the results are summarized in Table 4.6. We can recognize that the ratios of the $\nu_e$ event number ($N_{\nu_e,<0.06s}$) to the total event number of the charged-current reactions ($N_{\nu_e,<0.06s} + N_{\bar{\nu}_e,<0.06s}$) and that for the neutral-current reaction ($N_{NC,<0.06s}$) are larger for the models whose neutronization burst declines more steeply. Despite the fact that these neutronization burst numbers are of the order of 10, we can probe into the black hole progenitors in principle.
Chapter 5

Collapse of Population III Stars

5.1 Dynamical Features and Neutrino Signals

In this chapter, we summarize the numerical results of the gravitational collapse of the Pop III massive stars (models with $s \geq 15.98 k_B$; F1-F18). At first we present the result of the reference model with the initial mass $M_i = 375 M_\odot$ ($s = 17.5 k_B$; F2), whose density and temperature profiles are shown in Figure 5.1 and then we make comparison with different models. In this chapter, we always measure the time from the point at which the apparent horizon is formed.

Just like the ordinary collapse-driven supernova, the collapsing core is divided into two parts, to which we refer as the inner and outer cores. The inner core contracts subsonically and homologously ($U \propto r$) while the outer core infalls supersonically like free-fall ($U \propto r^{-1/2}$) except in the very late phase, in which the black hole formation is imminent. We can see this structure in Figure 5.2, in which we plot the radial velocity profiles at different times for the reference model. In Figure 5.3 we plot also the sound-speed profiles at $t = -1.52$ ms and $t = 0$ ms (the time of the apparent-horizon formation) for the same model. Around $t = 0$ ms, the homology does not hold any more due to the general relativistic strong gravity and the inner core splits into two parts. It is noted that the inner core is the main source of neutrinos especially in the late phase of collapse. This will be discussed later again.

In order to emphasize the importance of neutrinos for the dynamics, we make a comparison between the results with and without neutrino transport in Figure 5.4. Since the neutrino cooling is more efficient as the density increases, the neutrino emission accelerates the collapse in the central region in the early phase. As a result, the inner core becomes smaller when the
Figure 5.1: Density (left) and temperature (right) profiles for model F2 ($M_i = 375M_\odot$). The lines correspond, from bottom to top, to $t = -8.88$ s, $t = -239$ ms, $t = -41.6$ ms, $t = -12.3$ ms, $t = -1.52$ ms and $t = 0$ ms.

Figure 5.2: Profiles of the radial velocity for model F2 ($M_i = 375M_\odot$). The notation of lines is the same as in Figure 5.1.
neutrino transport is taken into account. The formation of apparent horizon is also affected. The baryon mass coordinate where the apparent horizon forms for the first time is $4.08M_{\odot}$ for the model with neutrino transport, whereas that of the model without neutrino transport is $13.2M_{\odot}$. Moreover, the time until the apparent horizon emerges becomes shorter. In Figure 5.5, we show the evolution of the entropy per baryon. For the model without neutrino transport, the dynamics is adiabatic and the entropy is constant except at the shock formed around the core surface ($\sim 60M_{\odot}$), where the entropy is generated. For the model with neutrino transport, on the other hand, the entropy per baryon decreases because of the neutrino cooling.

We now turn to the evolution of electron fraction ($Y_e$). As the electron capture proceeds, $Y_e$ is depleted as shown in Figure 5.6. Near the center, however, $Y_e$ then starts to increase, which is not seen in the ordinary supernova core. The main reasons are that the electrons are not degenerate due to the high entropy in the present model and that the reaction rates of electron capture and positron capture are different (e.g., Bruenn 1985). When the neutrino energy is lower than or comparable to the mass difference of proton and neutron, the electron capture dominates over the positron capture. As the neutrino energy becomes higher than the mass difference, these reaction rates become comparable, since electron-positron pairs are abundant. As a result, the electron capture is dominant and lowers $Y_e$ first, and the positron capture catches up afterward, leading to the rise of $Y_e$ before the complete
CHAPTER 5. COLLAPSE OF POPULATION III STARS

Figure 5.4: Comparison of the results without (left) and with (right) neutrinos for the same initial model with \( M_i = 375M_\odot \) (model F2). In each panel, the triangles show the locations of the apparent horizon. In the left panel, the lines correspond, from bottom to top, to \( t = -12.0 \text{ s}, t = -343 \text{ ms}, t = -42.3 \text{ ms}, t = -4.92 \text{ ms}, t = -0.699 \text{ ms} \) and \( t = 0 \text{ ms} \). The right panel is the same as the left panel in Figure 5.1.

Figure 5.5: The entropy profiles for model F2 \( (M_i = 375M_\odot) \) without (left) and with (right) neutrinos. The notation is the same as in Figure 5.4.
Figure 5.6: Profiles of $Y_e$ (left) and $Y_l$ (right) for model F2 ($M_i = 375M_\odot$). The notation of lines is the same as in Figure 5.1.

$\beta$-equilibrium is achieved. In Figure 5.6, we also show the evolution of lepton fraction, $Y_l$. Before neutrino trapping, the $Y_l$ profile evolves in the same way as the $Y_e$ profile. Then, $Y_l$ becomes unchanged and only $Y_e$ varies after neutrino trapping. We can confirm these behaviors in the neutrino luminosities given in Figure 5.7.

A large amount of neutrinos are emitted in the gravitational collapse of Pop III massive stars. The luminosity becomes as high as $\sim 10^{54}$ erg s$^{-1}$, much greater than the value for the ordinary supernova, $\sim 10^{53}$ erg s$^{-1}$. However, the total energy is $\sim 10^{53}$ erg, which is comparable to that of the ordinary supernova. This is because the apparent horizon is formed during the collapsing phase and, as a result, the neutrino emission lasts only for $\sim 100$ ms.

For the stars of current interest, the neutrino sphere is formed in general before the black hole formation. It is also noted that the apparent horizon is formed inside the neutrino sphere. In Figure 5.8, we show the neutrino-luminosity profiles and the location of the neutrino sphere for $\nu_e$ and $\bar{\nu}_e$ with different energies at $t = -12.3$ ms. At this moment, the inner-core surface is located at $r = 5 \times 10^7$ cm and we can recognize that the neutrino luminosity decreases towards the center inside the inner core. This is because the interactions of neutrinos with matter are frequent enough to make the neutrino angular distribution isotropic. In the outer core, on the other hand, neutrinos produced mainly inside the inner core are flowing out and the neutrino angular distribution is not isotropic. In our model, the outer core is...
Figure 5.7: Profiles of neutrino luminosity for model F2 ($M_t = 375M_\odot$). Upper left, upper right, lower left and lower right panels are for $\nu_e$, $\bar{\nu}_e$, $\nu_x$ and their sum, respectively, where $\nu_x$ stands for $\mu$- and $\tau$-neutrinos and anti-neutrinos. The notation of lines is the same as in Figure 5.1.
Figure 5.8: Profiles of the neutrino luminosity with different energies for model F2 ($M_i = 375M_\odot$) at $t = -12.3$ ms. Left and right panels are for $\nu_e$ and $\bar{\nu}_e$, respectively. In each panel, the dot-dashed line, short dashed line, solid line, long dashed line and short-long dashed line correspond to the neutrino luminosity with 2.51 MeV, 6.31 MeV, 15.8 MeV, 35.4 MeV and 70.7 MeV. The points on each line mark the locations of neutrino sphere for each energy.

as thick as $\sim 10^9$ cm and, as a result, the neutrino sphere is far away from the inner-core surface for high energy neutrinos. For instance, the optical depth of $\nu_e$ with 15.8 MeV is about 4 on the inner-core surface. These neutrinos are mainly emitted from the inner core and are absorbed or scattered by nucleons and electrons in the outer core. As a result, the luminosity decreases mildly as the neutrinos propagate from the inner core surface to the neutrino sphere. For the neutrinos with much higher energy, the reduction of the luminosity in the outer core is more remarkable because the optical depth is larger. On the contrary, the neutrino sphere for neutrinos with lower energy is close to or inside the inner core surface.

The luminosities of low energy neutrinos increase outside the neutrino spheres. Although these low energy neutrinos have ceased to react with matter, neutrinos with higher energy are still scattered down to lower energy, raising the luminosities of low energy neutrinos. This is also the reason why the mean energy of neutrinos is reduced substantially from the value at the inner-core surface as they propagate through the outer core to the stellar surface. Note that it is even lower than the mean energy for the ordinary supernova. Incidentally, in the outermost region ($\gtrsim 3 \times 10^8$ cm in Figure 5.8),
the neutrino luminosities decrease slightly. This reflects the difference of the emission times. As the collapse goes on, the matter become denser and hotter and the neutrino emission occurs more efficiently later on.

We also investigate the other neutrino species. As mentioned in SS 4.2.2, the differences between $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$ and $\bar{\nu}_\tau$ are ignored and we denote these four species as $\nu_x$ collectively. In Figure 5.9 we show the luminosity profiles and the locations of neutrino sphere for $\nu_x$ at $t = -12.3\,\text{ms}$ and $t = -1.52\,\text{ms}$. For $\nu_x$, the radii of the neutrino spheres are smaller than those of $\nu_e$ and $\bar{\nu}_e$, since $\nu_x$ do not have charged current reactions. Outside the neutrino sphere, the negative gradient of the luminosity is steeper for $\nu_x$ than for $\nu_e$ and $\bar{\nu}_e$, especially for high energy neutrinos. The absence of the charged current reactions makes the core optically thinner for $\nu_x$, and the rise in the temperature in the inner core is reflected more immediately by the neutrino luminosity for $\nu_x$.

It is already mentioned that the average energies of emitted neutrinos are rather lower than those for the ordinary supernova. The temporal evolutions of neutrino spectra at the outer boundary of our computation region are shown in Figure 5.10 for the reference model. The spectra after $t = 0\,\text{ms}$ are evaluated with the assumption that the neutrinos outside the neutrino sphere will flow freely. The last time in Figure 5.10, $t = 85.0\,\text{ms}$, is corresponding to the light-crossing time from the neutrino sphere (roughly the location of the filled circle in the right panel of Figure 5.9) to the surface. We can see
that the neutrino spectra get harder as the time passes. This is because the inner core becomes hotter as the collapse goes on.

We compare our results with those of Fryer et al. (2001). Their non-rotation model with an initial mass of $300M_{\odot}$ gives a black hole at the end. The oxygen core mass is $\sim 180M_{\odot}$ (see Figure 2 in their paper). We choose the reference model for comparison, since it has the same oxygen core mass $M_{O} \sim 180M_{\odot}$, though the total mass is a little bit larger, $M_{i} = 375M_{\odot}$.

The qualitative features are similar. For example, the collapsing iron core is divided into two parts, the inner and outer cores. The electron fraction ($Y_{e}$) starts to increase near the center after it decreases at first. The black hole is formed without bounce. However, the quantitative differences can be recognized also. The most outstanding one is the size of the inner core. In our model, it is $\sim 14M_{\odot}$ while it is $\sim 40M_{\odot}$ (see Figure 3 in their paper) in their model. We suspect for the moment that the main reason for the difference is the treatment of the neutrino transport. In their model, the neutrinos are treated in the so-called grey approximation, where only the energy-averaged distribution is employed. As demonstrated earlier, the location of neutrino sphere is highly energy-dependent. With only an average energy taken into account, the neutrino trapping will be overestimated. On the other hand, we treat multi-energy groups in our simulations, and the inner core can be cooled by low energy neutrinos. Due to this enhanced neutrino cooling, the inner core becomes smaller in our model. It is, however, noted that the difference of the initial models may be also important.

The total emitted energy also differs. We estimates it to be $\sim 4 \times 10^{53}$ erg in our model, which is an order of magnitude smaller than $\sim 3 \times 10^{54}$ erg in their model (see Figure 4 in their paper). The discrepancy seems to come from the employed criterion for the black hole formation. They assumed that the black hole is formed at the time when a certain fraction of the star falls within the last stable orbit, $r = 3r_{g}$ (e.g., Shapiro & Teukolsky 1983), while we utilized the apparent-horizon formation, which is a rigorous sufficient condition, as mentioned already. This difference leads to the substantial difference in the durations of neutrino emissions. In fact, the neutrino emission lasts for $\sim 100$ ms in our model, while their estimate is $\sim 1000$ ms (see Figure 4 in their paper). Note that even our estimate will be a bit overestimation, since the event horizon is located somewhat farther out from the apparent horizon.

Now we move on to the comparison between different models to see the initial-mass dependence of the dynamics and neutrino emissions. The qualitative features mentioned above, such as the split of the inner core or the increase of $Y_{e}$, are common to all the models. The central density, $\rho_{c}$, at the moment when the apparent horizon forms is smaller for more massive models,
Figure 5.10: Spectra of emitted neutrinos for model F2 ($M_i = 375 M_\odot$). Upper left, upper right, lower left and lower right panels are for $\nu_e$, $\bar{\nu}_e$, $\nu_x$ ($= \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$) and their sum, respectively. The dot-dashed line, solid line, short dashed line and long dashed line represent, respectively, the spectra at $t = -239 \text{ ms}$, $t = 0 \text{ ms}$, $t = 60.3 \text{ ms}$ and $t = 85.0 \text{ ms}$. 
Table 5.1: Initial Mass Dependence of Core Entropy, Core Mass and Location of Apparent Horizon.

<table>
<thead>
<tr>
<th>model</th>
<th>$M_i$ (M$_\odot$)</th>
<th>$s_{\text{iron}}$ (kB)</th>
<th>$s_{\text{core}}$ (kB)</th>
<th>$M_{\text{core}}$ (M$_\odot$)</th>
<th>$M_{\text{AH}}$ (M$_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>300</td>
<td>15.98</td>
<td>7.33</td>
<td>13.6</td>
<td>4.17</td>
</tr>
<tr>
<td>F2</td>
<td>375</td>
<td>17.50</td>
<td>7.37</td>
<td>14.1</td>
<td>4.08</td>
</tr>
<tr>
<td>F3</td>
<td>470</td>
<td>19.17</td>
<td>7.70</td>
<td>15.8</td>
<td>4.53</td>
</tr>
<tr>
<td>F4</td>
<td>585</td>
<td>20.96</td>
<td>7.88</td>
<td>16.8</td>
<td>4.84</td>
</tr>
<tr>
<td>F5</td>
<td>730</td>
<td>22.97</td>
<td>8.07</td>
<td>18.2</td>
<td>5.18</td>
</tr>
<tr>
<td>F6</td>
<td>915</td>
<td>25.25</td>
<td>8.28</td>
<td>19.5</td>
<td>5.39</td>
</tr>
<tr>
<td>F7</td>
<td>1145</td>
<td>27.77</td>
<td>8.50</td>
<td>21.0</td>
<td>5.82</td>
</tr>
<tr>
<td>F8</td>
<td>1430</td>
<td>30.54</td>
<td>8.77</td>
<td>23.4</td>
<td>6.33</td>
</tr>
<tr>
<td>F9</td>
<td>1800</td>
<td>33.74</td>
<td>8.90</td>
<td>24.7</td>
<td>6.46</td>
</tr>
<tr>
<td>F10</td>
<td>2250</td>
<td>37.19</td>
<td>9.24</td>
<td>26.5</td>
<td>6.96</td>
</tr>
<tr>
<td>F11</td>
<td>2800</td>
<td>40.96</td>
<td>9.64</td>
<td>30.0</td>
<td>7.60</td>
</tr>
<tr>
<td>F12</td>
<td>3500</td>
<td>45.24</td>
<td>10.10</td>
<td>33.5</td>
<td>8.42</td>
</tr>
<tr>
<td>F13</td>
<td>4350</td>
<td>49.89</td>
<td>10.35</td>
<td>36.0</td>
<td>8.44</td>
</tr>
<tr>
<td>F14</td>
<td>5500</td>
<td>55.50</td>
<td>10.56</td>
<td>39.0</td>
<td>9.11</td>
</tr>
<tr>
<td>F15</td>
<td>6800</td>
<td>61.16</td>
<td>10.41</td>
<td>37.6</td>
<td>9.10</td>
</tr>
<tr>
<td>F16</td>
<td>8500</td>
<td>67.78</td>
<td>10.58</td>
<td>40.1</td>
<td>9.40</td>
</tr>
<tr>
<td>F17</td>
<td>10500</td>
<td>74.75</td>
<td>10.48</td>
<td>39.5</td>
<td>9.26</td>
</tr>
<tr>
<td>F18</td>
<td>13500</td>
<td>84.06</td>
<td>11.20</td>
<td>44.5</td>
<td>9.91</td>
</tr>
</tbody>
</table>

$M_i$ and $s_{\text{iron}}$ are the same as in Table 4.2. $s_{\text{core}}$ is the entropy per baryon at the center of the inner core at $t = 0$. $M_{\text{core}}$ is the mass of the inner core at $t = 0$ and $M_{\text{AH}}$ is the location of the apparent horizon at $t = 0$.

which is similar to the results by Linke et al. (2001). For instance, we find that the central density is $\rho_c = 7.51 \times 10^{14}$ g cm$^{-3}$ for model F2 ($M_i = 375 M_\odot$) while it is $\rho_c = 1.23 \times 10^{14}$ g cm$^{-3}$ for model F17 ($M_i = 10,500 M_\odot$).

As the initial mass $M_i$ gets larger, the initial value of entropy per baryon in the iron core, $s_{\text{iron}}$, becomes greater. Our results show that the final value of entropy per baryon at the center of core, $s_{\text{core}}$, does not become larger proportionately. For massive models, in fact, $s_{\text{core}}$ is saturated. The reason is that the neutrino cooling is more efficient for more massive stars, since electrons are non-degenerate and a large amount of electron-positron pairs exist. For instance, the ratio of the net electron number to the sum of the numbers of electron and positron at the center in the initial model is 27.6% for model F2 ($M_i = 375 M_\odot$) while it is 5.2% for model F17 ($M_i = 10,500 M_\odot$).
The existence of a large amount of electron-positron pairs leads to a smaller difference between electron- and positron-captures. This is reflected on the minimum value of $Y_e$ at the center, which is 0.197 in model F2 and 0.235 in model F17. Owing to the very efficient neutrino cooling, the inner-core mass, $M_{\text{core}}$, and the location of the apparent horizon, $M_{\text{AH}}$, at $t = 0$ ms do not increase with the initial mass, $M_i$, either. These results are summarized in Table 5.1. For very massive stars, the inner-core fraction gets smaller and the general relativistic strong gravity pulls not only the vicinity of the center but also the entire inner core. This makes the substructure of the inner core indiscernible by $t = 0$ ms (see, for example, the model with $M_i = 10,500 M_\odot$ in Figure 5.11).

The radius of the inner core, which is $\sim 3 \times 10^7$ cm for model F2 and $\sim 5 \times 10^7$ cm for model F17, is also insensitive to the initial mass, $M_i$. However, the radius of the neutrino sphere depends on $M_i$. For instance, the radius of the neutrino sphere for $\nu_e$ with 15.8 MeV at $t = 0$ ms is $\sim 1.5 \times 10^8$ cm for model F2 with $M_i = 375 M_\odot$ while it is $\sim 10^9$ cm for model F17 with $M_i = 10,500 M_\odot$. This is mainly because more massive stars have thicker outer cores, which are opaque to neutrinos.

In Figure 5.12 we plot the time-integrated energy and number spectra of neutrinos for 4 different models based on the upper limit discussed above. As expected, the more massive the initial mass is, the larger the total emission energy is, since the liberated gravitational energy is greater. It is interesting, however, that the spectra do not become harder but rather softer as the...
Figure 5.12: Neutrino energy (left) and number (right) spectra. The dot-dashed line, short dashed line, long dashed line and solid line represent models F2 ($M_i = 375 M_\odot$), F7 ($M_i = 1145 M_\odot$), F12 ($M_i = 3500 M_\odot$) and F17 ($M_i = 10,500 M_\odot$), respectively.

initial mass increases. There are two reasons. The first reason is again that the neutrino cooling is more efficient for more massive stars. As a result, the physical condition in the inner core, the main source of neutrinos, is similar among different models. The second reason, which is also mentioned already, is that massive models have a very thick outer core, which prevents high energy neutrinos from getting out of the inner core directly. As seen in Table 4.4, this is particularly remarkable for $\nu_x$, the species with the lowest reaction rates. The reason is as follows. The distance from the stellar surface down to the neutrino sphere is almost independent of the initial mass. On the other hand, the distance from the stellar surface to the inner-core surface, is dependent on it. As the initial mass increases, so does the distance between the inner-core surface and the neutrino sphere. Since the neutrino sphere for $\nu_x$ is inside those for $\nu_e$ and $\bar{\nu}_e$ and the temperature scale height is smaller near the inner core, the average energy of $\nu_x$ is more sensitive to the difference of the initial mass.

5.2 Relic Neutrinos from Population III Stars

In this section, we estimate the number flux of diffuse relic neutrinos from Pop III massive stars, as already done for ordinary supernovae (e.g., Ando &
Sato 2004; Strigari et al. 2004). After giving the formulation, we apply it to several models of the star formation history and discuss possible constraints on them. In the following, we do not take into account the possible mixing of neutrinos in the stellar envelope, since it depends on a detailed density profile of the outer envelope, which we do not have at hand. See Appendix B for the details.

### 5.2.1 Formulation for Relic Neutrino Flux

Here we formulate the expression for the number flux of the relic neutrino from Pop III massive stars. We assume that Pop III stars in the mass interval, $M_0 \leq m \leq M_N$, collapse and emit neutrinos at redshift $z_i \geq z \geq z_f$. Then the present number flux of relic neutrinos on the earth is given by

$$dF_\nu/dE_\nu = c \int_{z_i}^{z_f} \int_{M_0}^{M_N} dN(m, E'_\nu)/(1+z) R_{\text{PopIII}}(z, m) \frac{dt}{dz} dz,$$

where $E_\nu$ is the detected neutrino energy and $E'_\nu = (1+z)E_\nu$ is the emitted neutrino energy and $c$ is the velocity of light. The neutrino number spectrum emitted by the progenitor with mass, $m$, is $dN(m, E'_\nu)/dE'_\nu$. We denote the birth rate of Pop III massive stars per comoving volume per mass as $R_{\text{PopIII}}(z, m)$. Massive stars are supposed to die immediately after the birth. The relation between $t$ and $z$ is given by

$$\frac{dz}{dt} = -H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda},$$

and the cosmological parameters are given as $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 71^{+4}_{-3}$ km/s/Mpc, according to the standard ΛCDM cosmology (Spergel et al. 2003).

Here we assume that $R_{\text{PopIII}}(z, m)$ can be written as

$$R_{\text{PopIII}}(z, m) dm dz = \frac{dn(m)}{dm} dm \psi(z) dz = dn(m) \psi(z) dz,$$

where $dn(m)$ is the number of Pop III massive stars within the mass interval, $[m, m + dm]$. The normalization factor $\psi(z)$ is chosen to satisfy the following condition,

$$\int_{z_i}^{z_f} \psi(z) \frac{dz}{dz} dz = \int_{z_i}^{z_f} \psi(z) H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} dz = 1.$$

Thus we can rewrite equation (5.1) as

$$dF_\nu/dE_\nu = c \int_{M_0}^{M_N} dm \frac{dn(m)}{dm} \int_{z_i}^{z_f} dz \frac{\psi(z)}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} dN(m, E'_\nu)/(1+z).$$

(5.5)
In the above equation, \( \frac{dn}{dm} \) represents the initial mass function (IMF). Here we adopt the IMF of Pop III stars proposed by Nakamura & Umemura (2001). Their IMF is bimodal and the heavier component is given as

\[
\begin{cases}
\frac{dn}{dm} = Bm^{-\beta-1} & \text{for } m \geq M_{\min}, \\
n = 0 & \text{for } m < M_{\min},
\end{cases}
\]

(5.6)

where \( \beta > 1 \) and \( B > 0 \) are independent of \( m \). Incidentally, the IMF given by Salpeter (1955) has \( \beta = 1.35 \) for the mass range, \( 0.4M_\odot < m < 10M_\odot \).

The mass density of Pop III massive stars is given as

\[
P_{\text{all}} = \int_{M_{\min}}^{\infty} m dn = \frac{B}{\beta - 1} M_{\min}^{1-\beta}.
\]

(5.7)

On the other hand, we denote the time-integrated fraction of cosmic baryon converted into Pop III stars as \( \epsilon \) and the fraction of the mass contained in the heavier population as \( (1 - \kappa) \). Then \( P_{\text{all}} \) can be also expressed as

\[
P_{\text{all}} = n_b \, m_N \, \epsilon \, (1 - \kappa),
\]

(5.8)

where \( n_b = (2.5 \pm 0.1) \times 10^{-7} \text{ cm}^{-3} \) is the present number density of cosmic baryon (Spergel et al. 2003) and \( m_N \) is the nucleon mass. From equations (5.7) and (5.8), we can determine the normalization factor \( B \) as

\[
B = (\beta - 1) M_{\min}^{\beta-1} n_b m_N \epsilon (1 - \kappa).
\]

(5.9)

In order to evaluate the relic neutrino flux from our models, we approximate the integral in equation (5.5) by the summation over mass bins covering the mass range of our models. Using the equation

\[
\int_{M_{k-1}}^{M_k} dm \frac{dn(m)}{dm} = \frac{\beta - 1}{\beta} n_b m_N \epsilon (1 - \kappa) M_{\min}^{\beta-1} \left( M_{k-1}^{-\beta} - M_k^{-\beta} \right),
\]

(5.10)

we rewrite equation (5.5) as

\[
\frac{dF_\nu}{dE_\nu} = \frac{\beta - 1}{\beta} c n_b m_N \epsilon (1 - \kappa) M_{\min}^{\beta-1} \sum_{k=1}^{N} \left( M_{k-1}^{-\beta} - M_k^{-\beta} \right) \times \int_{z_i}^{z_f} dz \frac{\psi(z)}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \frac{dN(M_k, E'_\nu)}{dE'_\nu}(1+z),
\]

(5.11)

where \( \tilde{M}_k \) is approximated by the geometric mean of \( M_{k-1} \) and \( M_k \) and is chosen to coincide with the mass of our models. \( M_k \) is independent of \( \beta \).
and given in Table 5.2. This approximation is estimated to give at most several percents of error for the range of $1 < \beta < 3$. Here we use 18 mass bins starting from $M_0 = 260\, M_\odot$, the largest mass for the pair-instability supernova, to $M_{18} = 16,000\, M_\odot$, the smallest mass for the onset of collapse by the general relativistic effect (§4.1.2).

In the following, we take the most optimistic values for the uncertain quantities in estimating the relic neutrinos. For the total neutrino emissions, we take the upper limit given in Table 4.4. In the IMF by Nakamura & Umemura (2001), we choose $(1 - \kappa) = 1$. Since they estimate that $M_{\text{min}}$ is in the range of a few times $10^{-100} \, M_\odot$, we take $M_{\text{min}} = 100 \, M_\odot$. The formation efficiency of Pop III stars, $\epsilon$, one of the most uncertain factors, is suggested to be rather large, of the order of $10\%$, from the estimations of the contribution of Pop III stars to the cosmic infrared background (e.g., Santos et al. 2002; Salvaterra & Ferrara 2003; however, see also Madau & Silk 2005). Here we assume $\epsilon = 0.1$. The change of the relic neutrino flux due to the variations of $(1 - \kappa)$, $M_{\text{min}}$ or $\epsilon$ is obtained from equation (5.11). We find that these parameters do not affect the peak energy of neutrino. $\beta$ is also an uncertain parameter, and its influence on the flux is complicated. Hence we take several values.

As for the star formation history, $\psi(z)$, we employ the following three models. The observation by WMAP suggests that the reionization occurred at the redshift of $z = 17 \pm 5$ (Spergel et al. 2003). Based on this, we assume the following function for $\psi(z)$ as model A.

$$
\psi(z) = \psi_A(z) \equiv \frac{1}{5\sqrt{2\pi}} \exp \left( -\frac{(z - 17)^2}{20} \right) H_0 (1 + z) \sqrt{ \Omega_m (1 + z)^3 + \Omega_A }.
$$

On the other hand, according to the theoretical investigation of the star formation history by Scannapieco et al. (2003), the peak of the formation of Pop III stars might have been at $z \sim 10$. Hence, as model B, we take

$$
\psi(z) = \psi_B(z) \equiv \delta(z - 10) H_0 (1 + z) \sqrt{ \Omega_m (1 + z)^3 + \Omega_A }.
$$

Some authors have attempted to estimate the formation rate of Pop III stars
5.2.2 Results and Discussion

In Figure 5.13, we plot the relic neutrino fluxes for model A with $\beta = 1.35$. It is seen that the peak energy of $\nu_x$ ($= \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$) is smaller than $\nu_e$ and $\bar{\nu}_e$. At first glance, this seems inconsistent with the trend of the average energies shown in Table 4.4. However, the number flux of $\nu_e$ has a high energy tail because of the absence of charged current reactions. Integrated over the spectrum, the mean energy of $\nu_x$ becomes higher than those of $\nu_e$ and $\bar{\nu}_e$ as given in Table 4.4.

We show in the left panel of Figure 5.14 the relic $\bar{\nu}_e$ fluxes for different values of $\beta$. We find that the peak energy is not sensitive to $\beta$. This is due to the fact that the mean energy of the neutrinos is insensitive to the initial stellar mass, which we have already mentioned. We also find that the model with $\beta = 1.35$ gives the greatest number flux. This can be understood as
Figure 5.14: Relic $\bar{\nu}_e$ number fluxes from Pop III massive stars for various values of $\beta$ and different star formation histories. The left panel shows the result of model A and the short dashed line, solid line, long dashed line and dot-dashed line correspond to $\beta = 1.1$, $\beta = 1.35$, $\beta = 2$ and $\beta = 3$, respectively. The right panel shows the result for $\beta = 1.35$ and the solid line, long dashed line, and short dashed line represent models A, B and C, respectively.

follows. We first remind readers that the total mass density, $P_{\text{all}}$, is fixed when we vary $\beta$ (see equations (5.7) and (5.8)). If $\beta$ is close to unity, a substantial fraction of stars falls in the realm of supermassive stars ($\gtrsim 16,000 M_\odot$), which we ignore in this paper. For larger $\beta$, on the other hand, a large population of stars contribute to the pair-instability supernovae ($\lesssim 260 M_\odot$), which are not considered here, either. In the right panel of Figure 5.14 we make a comparison of the different models of the star formation history. It is found that the larger the redshift at which Pop III massive stars are formed is, the lower the peak energy becomes. We have seen that the peak energy is not sensitive to IMF. Hence the peak energy is determined solely by the redshift of the Pop III star formation. It is noted, however, that the neutrino oscillation is neglected in these calculations. Since the average energies for all species of neutrinos are insensitive to IMF, we expect that the peak energy will be still insensitive to IMF after the neutrino mixing. In order to see its effect quantitatively, however, detailed density profiles of stellar envelope are needed, and we defer the investigation to the future work. Iocco et al. (2005) presented the spectrum that is harder than that we have obtained in this paper. This is simply because, they employed the results by Fryer et al.
(2001), which predicted higher neutrino energies than our results.

As for the possibilities of detection, we must say that it is difficult for the currently operating detectors. In spite of the above-mentioned difference of our results and that of Iocco et al. (2005), both results are negative for detection, because the cosmological redshift reduces the peak energy too much. To put it more precisely, the relic $\nu_e$ fluxes from Pop III massive stars are overwhelmed by solar $\nu_e$ below 18 MeV and by the relic neutrinos from ordinary supernovae above $\sim 10$ MeV. As for $\bar{\nu}_e$, the emissions from nuclear reactors are the main obstacle below 10 MeV. Thus the existing detectors can not distinguish Pop III relic neutrinos from others. However, because the solar and reactor neutrinos are not isotropic, removing them is possible at least in principle. For $\bar{\nu}_e$, in particular, Pop III massive stars are the largest cosmological source. In the future, we may be able to discuss the formation history of Pop III stars with these diffuse neutrino fluxes.
Chapter 6

Collapse Including Quarks and Pions

In this chapter, we perform numerical simulations for the gravitational collapse of the massive stars using the EOS’s constructed in §3.2. In the computations, we employ different models, in part, from the preceding chapters. Hereafter, we set the bag constant $B = 250$ MeV fm$^{-3}$ for all the models with quarks. It is noted that this value of the bag constant can reproduce some desired properties of hot and dense matter as discussed in §3.3.

6.1 Initial Models

In Table 6.1 the models adopted in this chapter are listed. The model with $375M_\odot$ (model Z) is same with the model F2 in Chapter 5 and its results of the collapse under the Shen EOS are already stated. The model with $40M_\odot$ (model X) is a result of evolutionary calculations for Pop I stars while the mass loss is not taken into account (Woosley & Weaver 1995). Incidentally, the numerical simulation of its collapse has been already done under the Shen EOS by Sumiyoshi et al. (2007) and their numerics is almost same with ours. According to them, the core has a bounce by the nuclear force as in the case of ordinary supernovae and recollapse to the black hole at 1.35 sec after the bounce. Model Y is a realistic stellar model of $100M_\odot$ constructed by Nomoto et al. (2005) with evolutionary calculations. This model is supposedly a Pop III star and we use this model as a reference model in this chapter. Before the discussions of the quarks and pions, we describe the collapse of model Y under the Shen EOS.

The entropy at the center of model Y is higher than that of ordinary supernova progenitors when it starts to collapse because the star does not


Table 6.1: Key Parameters for the Models in this Chapter

<table>
<thead>
<tr>
<th>model</th>
<th>$M_{\text{total}}$ ($M_\odot$)</th>
<th>reference</th>
<th>$s_{\text{initial}}$ ($k_B$)</th>
<th>$M_{\text{iron}}$ ($M_\odot$)</th>
<th>$t_{\text{recollapse}}$ (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>40</td>
<td>Woosley &amp; Weaver (1995)</td>
<td>0.85</td>
<td>1.98</td>
<td>1345</td>
</tr>
<tr>
<td>Y</td>
<td>100</td>
<td>Nomoto et al. (2005)</td>
<td>3.50</td>
<td>2.32</td>
<td>402</td>
</tr>
<tr>
<td>Z</td>
<td>375</td>
<td>this thesis</td>
<td>17.5</td>
<td>65.0</td>
<td>---</td>
</tr>
</tbody>
</table>

$M_{\text{total}}$, $s_{\text{initial}}$ and $M_{\text{iron}}$ denote the total mass of the star, the entropy per baryon at the center of iron core and the mass of iron core, respectively.

lose its mass at all in its evolution owing to its zero metallicity. Here we relate model Y with the isentropic models investigated in the preceding chapters. In Figure 6.1, we show the comparison of the initial state of model Y and the isentropic models at the time when their central densities become the same as that of model Y. We can recognize that model Y has the entropy $\sim 3.5k_B$ in the central region, which is between those of model E1 and E2, whereas the iron core model of Y is smaller than that of isentropic models. In fact, the iron core mass of model Y is $\sim 2.32M_\odot$, which is close to that of model E1. Incidentally, the initial velocity profile is taken into account for model Y although it is much lower than the sound speed at each point.

As a result of collapse, model Y has a bounce and recollapses to a black hole. The central density and the central adiabatic index of model Y at the bounce are $1.37 \times 10^{14}$ g cm$^{-3}$ and 2.19, respectively, which are between those of models E1 and E2. This suggests that the bounce mechanism is same with them and different from model X. On the other hand, model Y has a much longer time interval from the bounce to the recollapse, compared with models E1 and E2. This is because the inner core mass of model Y at the bounce $M_{\text{bounce}} = 0.65M_\odot$ is smaller than those of the isentropic models and the lower density of the outer core (Figure 6.1) gives lower accretion rates. For instance, at $t = 0.06$ s, model Y has a mass accretion rate $\sim 4M_\odot$ s$^{-1}$ at the shock surface whereas model E1 has $\sim 11M_\odot$ s$^{-1}$. Thus it takes much time until the inner core mass exceeds the maximum mass of the neutron star.

The time evolutions of the emitted neutrino luminosity are shown in Figure 6.2. The neutrino luminosity of model Y is lower than those of isentropic models, despite the fact that the total energy of emitted neutrinos from model Y, $E_{\text{all}} = 3.49 \times 10^{52}$ ergs is larger than those from isentropic models. This is because the neutrino emission of model Y lasts much longer. The mean energies of the emitted neutrinos for model Y, $\langle E_{\nu_e} \rangle = 15.34$ MeV,
Figure 6.1: Comparisons of the density profiles (left) and the entropy profiles (right). Solid lines represent the initial profiles for model Y and dashed and dot-dashed lines represent the profiles for models E1 and E2, respectively, at the time when the central density becomes the same as the initial central density of model Y.

$\langle E_{\bar{\nu}_e} \rangle = 18.90 \text{ MeV}$ and $\langle E_{\nu_x} \rangle = 23.42 \text{ MeV}$, are higher and this is also due to the longer duration time. The neutrino spectrum gets harder in the late phase because the density of the accreting matter becomes lower and the temperature on the neutrino sphere gets higher. Thus, the longer the duration time of neutrino emission is, the larger the mean energy of the emitted neutrinos becomes.

In the following, we discuss the features of the emitted neutrinos from model Y for the early phase, which is not sensitive to the EOS as already mentioned. Comparing Figures 4.5 and 6.2, we can see that for model Y, the peak luminosity of the electron-type neutrino by the neutronization burst is lower than those of isentropic models. The reason why it is lower than that of model E1 ($s = 3k_B$) is because the chemical potential of an electron-type neutrino for model Y is lower than that for model E1, while the shock radii in both models are not so different from each other (right panels of Figure 6.2). It is consistent with the fact that the density at the bounce of model Y is lower than that of model E1 (Tables 4.3 and 6.1). On the other hand, the shock radii of models with $s \geq 4k_B$ (models E2-E4) are larger than that of model Y. This is the reason why the luminosity of the neutronization burst for model Y is lower than those of models E2-E4. Furthermore, since the
Figure 6.2: Results of the collapse for model Y. In the left and right panel, the notations of lines are the same as in Figures 4.5 and 4.6, respectively, but the end points of the lines in the left panel represent the time when the apparent horizon is formed.

6.2 Results and Discussions

In this section, we study the stellar collapse including quarks and pions. The investigations of the model Y (the reference model) are shown at first and we discuss the model dependence of the results successively. In the following, we use the same notations with §3.3.2, namely, EOS’s without pions and quarks (the original Shen EOS in §3.1), without pions but with quarks, with pions and without quarks and the model with pions and quarks are denoted as OO, OQ, PO and PQ, respectively.
Table 6.2: EOS Dependence of the Results for Model Y

<table>
<thead>
<tr>
<th>EOS</th>
<th>$t_{\text{recoll}}$ (msec)</th>
<th>$E_{\nu_e}$ (ergs)</th>
<th>$E_{\bar{\nu}_e}$ (ergs)</th>
<th>$E_{\nu_x}$ (ergs)</th>
<th>$E_{\text{all}}$ (ergs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>OO</td>
<td>402</td>
<td>$9.42 \times 10^{52}$</td>
<td>$7.89 \times 10^{52}$</td>
<td>$4.40 \times 10^{52}$</td>
<td>$34.9 \times 10^{52}$</td>
</tr>
<tr>
<td>OQ</td>
<td>352</td>
<td>$8.03 \times 10^{52}$</td>
<td>$6.58 \times 10^{52}$</td>
<td>$3.44 \times 10^{52}$</td>
<td>$28.4 \times 10^{52}$</td>
</tr>
<tr>
<td>PO</td>
<td>351</td>
<td>$8.05 \times 10^{52}$</td>
<td>$6.63 \times 10^{52}$</td>
<td>$3.67 \times 10^{52}$</td>
<td>$29.3 \times 10^{52}$</td>
</tr>
<tr>
<td>PQ</td>
<td>336</td>
<td>$7.60 \times 10^{52}$</td>
<td>$6.22 \times 10^{52}$</td>
<td>$3.34 \times 10^{52}$</td>
<td>$27.2 \times 10^{52}$</td>
</tr>
</tbody>
</table>

The definition of the EOS’s is given in the text. $t_{\text{recoll}}$ represents the interval time from the bounce to the apparent horizon formation. $E_{\nu_i}$ is the total energy of emitted $\nu_i$, where $E_{\nu_x} = E_{\nu_\mu} = E_{\bar{\nu}_\mu} = E_{\nu_\tau} = E_{\bar{\nu}_\tau}$. $E_{\text{all}}$ is the total energy summed over all species.

To begin with, we should modify the methods to compute the neutrino distribution functions. When we construct the EOS, we have assumed that the electron-type neutrinos are in equilibrium with other particles in the hadron-quark mixed phase and the pure quark phase. Hence after the phase transition occurs, we do not compute the neutrino distribution functions and assume that they are Fermi-Dirac functions for all species conserving the electron-type lepton fraction, $Y_l$, for each fluid element. Moreover, we neglect the entropy variation from the neutrino transport. We can justify this gap in the neutrino treatments because the density is high enough for neutrinos to be trapped anyway at the phase transition. Furthermore the diffusion time scale of neutrinos, $\sim 10$ s, is much longer than the physical time of the computation, which is the interval from the phase transition to the black hole formation.

In Figure 6.3 we show the time profiles of the central baryon mass density and neutrino luminosities. As stated in the preceding section, model Y have a bounce owing to thermal nucleons at subnuclear density ($\sim 1.4 \times 10^{14}$ g cm$^{-3}$) and then recollapse to a black hole. The contribution of pions makes a difference at $\sim 2 \times 10^{14}$ g cm$^{-3}$ and that of quarks does so for larger density. We can also see that the effect of quarks begins to work suddenly at the transition density. On the other hand, the effect of pions begins to work gradually because the thermal pions appear before the pion condensation. These features of them are also reflected in neutrino luminosities. We note that the duration of neutrino emission is almost the same as the interval time from the bounce to the apparent horizon formation. In Table 6.2 we show them for all EOS’s computed here. Since the EOS becomes softer owing to the contribution of pions and quarks, the interval time becomes shorter. The model by EOS OO (the original Shen EOS) takes 20% longer to recollapse...
Figure 6.3: Time profiles of the central baryon mass density (upper left) and luminosities of $\nu_e$ (upper right), $\bar{\nu}_e$ (lower left) and $\nu_x$ (lower right) for model Y, where $\nu_x = \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$. Thin dashed, thick dashed, thin solid and thick solid lines correspond to the EOS’s OO, OQ, PO and PQ, respectively. The time is measured from the point at bounce.
than EOS PQ does. Using this difference of the interval time, we may be able to probe observationally the EOS of hot dense matter in future.

Features discussed above can be seen in Figure 6.4. The profiles of the particle fractions and the baryon mass density of model Y with EOS PQ for each step are shown in this figure. The time when the central density is close to the nuclear density, $\rho_0 \sim 2.5 \times 10^{14}$ g cm$^{-3}$, corresponds to 214 ms after the bounce and 122 ms before the black hole (apparent horizon) formation. At this time, the population of thermal pions is minor and they do not affect the dynamics at all. We can confirm it also from the comparison of the EOS OQ and EOS PQ in Figure 6.3. When the central density is $\sim 2\rho_0$ (304 ms after the bounce and 32 ms before the black hole formation), the pion condensation has already occurred in the central region, however, quarks do not appear yet. When the central density is $\sim 10\rho_0$ (0.08 ms before the black hole formation), quarks occupy the central region. At this phase, the star is already collapsing dynamically to the black hole, which is consistent with the fact that the effect of quarks begins to work suddenly. At the time of the black hole formation, the central density grows to $1.6 \times 10^{16}$ g cm$^{-3}$ and the pure quark matter resides in the central region. Incidentally, there are nuclei and $\alpha$-particles in the outer region where nucleons and quarks do not exist.

The evolution of the central density and temperature is plotted with the phase diagram in Figure 6.5. In this diagram, the electron-type lepton fraction is fixed to $Y_l = 0.17$, which is same with the value at the center. From this figure we can see that the temperature decreases in the mixed phase despite the density increases. It looks unfamiliar, however, we can interpret it in the context of phase transition (Müller 1997). Here we assumed that the transition is a first-order while some authors regard it as second-order or cross-over (e.g., Aoki et al. 2006a). In the first-order transition, the release of latent heat occurs. It is noted that, in this case, the low-density (hadron) phase has the lower entropy, which is opposite to ordinary phase transition. On the other hand, matter in the collapsing star is compressed adiabatically. Therefore, the entropy does not vary and the temperature decreases.

We now turn to the neutrino emission from model Y. The time profiles of neutrino luminosities are plotted in Figure 6.6 and total neutrino energies emitted from each model are shown in Table 6.2. Comparing EOS’s OO and OQ or EOS’s PO and PQ, the neutrino luminosities do not differ each other because, again, quarks begins to work suddenly at the transition density. As a result, the total energies of emitted neutrinos for the models including quarks become lower than those of the models without quarks because the durations of the neutrino emission become shorter.

This trend can be seen in the effects of pions also. However, as seen in
Figure 6.4: Profiles of the particle fractions (upper plots) and the baryon mass density (lower plots) of model Y with EOS PQ, where $Y_i \equiv \frac{n_i}{n_B}$. $n_i$ represents the number density of the particle $i$ and $n_B$ does the baryon number density. Upper left, upper right and lower left panels correspond to 122 ms, 32 ms and 0.08 ms before the black hole (apparent horizon) formation whereas right panel does to the moment of the black hole formation. The region inside the vertical line in upper right panel represents where the pion condensation occurs. The vertical line of the lower plots in lower right panel represents the location of the apparent horizon.
Figure 6.5: The evolution of the central density and temperature of model Y with EOS PQ (thick solid line) and the phase diagram for $Y_l = 0.17$. Dot-dashed line represents boundary of hadronic matter and mixed matter and dashed lines does that of mixed matter and quark matter.

Figure 6.3 the neutrino luminosity for the models with pions differs gradually in time from that for the models without pions because the effect of pions works gradually. The neutrino luminosity summed over all species is given approximately by the accretion luminosity $L_{\nu}^{\text{acc}} \sim GM_{\nu}\dot{M}/R_{\nu}$ (Thompson et al. 2003), where $G$, $R_{\nu}$, $\dot{M}$ and $M_{\nu}$ are the gravitational constant, the radius of the neutrino sphere, the mass accretion rate and the mass enclosed by $R_{\nu}$, respectively. We can roughly regard the neutrino sphere as the surface on which neutrinos are emitted. Incidentally in our case, as is the case for an ordinary supernova, the proto-neutron star is formed before the black hole formation, and neutrinos are emitted mainly on the surface of the proto-neutron star. Thus we can regard $R_{\nu}$ and $M_{\nu}$ as the radius and mass of the proto-neutron star, respectively. On the other hand, the density of the proto-neutron star of the model with pions is greater than that of the model without pions comparing at the same time because the EOS becomes soft under the influence of pions as seen in Figure 6.3. Thus the radius of the proto-neutron star becomes smaller without changing the mass of it. As a result, the neutrino luminosity gets higher for the models including pions. While a simple comparison cannot be done because the quarks also affect the neutrino emission slightly, we can see that the model of EOS PO has higher total energy than that of EOS OQ although the interval times are almost the
same. In conclusion, pions make the total energy of the emitted neutrinos lower by shortening the interval time and higher by increasing luminosity.

Finally, we investigate the mass dependence of the collapse with the hadron-quark phase transition. In Figure 6.6 the time profiles of the central baryon mass density are shown for models X and Z. The results of model X are very similar to those of model Y, while their mechanism of bounces are different. For instance, the interval time from the bounce to the apparent horizon formation becomes about 20\% shorter for model X with EOS OQ comparing to the case of EOS OO, and it is 1.111 s. Therefore, the black hole progenitors which have a bounce by the nuclear force may be also utilized to probe observationally the EOS of hot dense matter. On the other hand, the phase transition makes no difference for the dynamics at all in model Z. This is because it does not have a bounce and collapse to a black hole directly. Owing to the general relativistic strong gravity, the star has already unstable to collapse when the quarks appear. Thus it is confirmed that the analyses for Pop III stars given in Chapter 5 are not changed at all.
Chapter 7

Conclusions

In this thesis, we have numerically studied gravitational collapse and black hole formation of very massive stars and evaluated the amount of emitted neutrinos in spherical models. In our computations, dynamics are followed until a black hole (apparent horizon) formation. Here we have taken into account the reactions and transports of neutrinos in detail for 4 species, namely, \( \nu_e, \bar{\nu}_e, \nu_\mu \) and \( \bar{\nu}_\mu \). \( \nu_\tau \) and \( \bar{\nu}_\tau \) are assumed to be the same as \( \nu_\mu \) and \( \bar{\nu}_\mu \), respectively. We have utilized realistic equations of state. In the early stage of this study, the equation of state by Shen et al (1998a, 1998b) is adopted, however, the stellar collapse under the equations of state constructed by ourselves is also shown later.

At first, we have performed systematic computations for the iron core mass of stars which result in the black hole formation. Investigated iron core mass range is very broad for \( \geq 3M_\odot \) and had not been studied well so far. As a result, unexpectedly, massive iron cores with \( \sim 10M_\odot \) produce a bounce owing to thermal nucleons, following which they collapse to black holes when the maximum mass is reached. As for the emitted neutrinos, the spectra of \( \bar{\nu}_e \) and \( \nu_x (= \nu_\mu, \bar{\nu}_\mu, \nu_\tau \) and \( \bar{\nu}_\tau \) become softer for more massive models, or higher entropy models, because a high entropy generates a large number of electron-positron pairs, which create \( \bar{\nu}_e \) and \( \nu_x \). The neutronization burst from more massive iron core becomes less remarkable or disappears completely. This is because the density at the bounce is lower and even the \( \nu_e \) number density in equilibrium becomes lower. If the initial velocity is lower than the sound speed, it does not affect the collapse very much. Moreover, assuming our models as the progenitors of intermediate mass black holes collapsing at the Galactic center, we have estimated the neutrino event numbers. As a result, for Super Kamiokande III, the ratio of the \( \bar{\nu}_e \) event number for \( < 10 \text{ MeV} \) to that for all events gets larger as the entropy of the core becomes higher, especially for \( 7.5k_B \leq s \leq 13k_B \). We have suggested that we can use these
features to probe into the progenitors. As for the lower entropy cores, despite the fact that the event number for the early phase of the emission is less than 100 by SNO, we have suggested that the steep decline of the neutronization burst can be distinguished in principle.

Some models investigated above with larger masses correspond to the Population III massive stars. We have found that they collapse to the black hole without bounce. Also in this case, neutrinos affect the dynamics of collapse crucially, determining the inner core mass, the location and formation time of the apparent horizon. Incidentally, it gives rise to the non-monotonic evolution of the electron fraction during the collapse, which is not seen in the ordinary supernova core. Neutrino cooling is very efficient owing to abundant electron-positron pairs. Then the final value of core entropy is not sensitive to the initial value or the initial stellar mass. For these massive stars, even the outer core becomes opaque to neutrinos. As a result, the neutrino spectra do not become harder as the iron core mass increases. Moreover, we have evaluated the flux of relic neutrinos from Population III massive stars. Since neutrino spectra do not depend on the stellar mass very much, the peak energy of relic neutrinos is mainly determined by the redshift of the Population III star formation and not sensitive to the initial mass function. We have also found that, unfortunately, the detection of these relic neutrinos is highly difficult at present.

Furthermore, we have constructed the equations of state including quarks and pions because the density and temperature of black hole progenitor get enough high for them to appear. Especially, the appearance of quarks corresponds to the QCD transition and we have treated it using Gibbs conditions for finite temperature. As for the pions, we have added them to the equation of state by Shen et al (1998a, 1998b), which is based on the relativistic mean field theory. Our results are found to reproduce the composition in the previous studies particularly for the pion population in the neutrino-less $\beta$ equilibrium state at zero temperature. We have adopted the MIT bag model of the deconfined 3-flavor strange quark matter for the equation of state of the quark phase. In addition, we have assumed that the equilibrium is achieved not only by the strong interactions but also by the weak interactions in the mixed phase. Incidentally, electrons and electron-type neutrinos are taken into account as being in weak equilibrium for the mixed phase and pure quark phase and muons are also treated but approximately. The resultant equations of state are confirmed to be thermodynamically stable and exhibit qualitatively the desired properties of hadron-quark mixed matter, such as the temperature dependence of the transition density. In particular, we have also found that the models with the bag constant $B \gtrsim 250$ MeV fm$^{-3}$ are favored quantitatively from the features in high temperature regime and the
maximum mass of compact stars.

Finally, we have computed the collapse of a massive star with $40M_\odot$, $100M_\odot$ and $375M_\odot$ using equations of state constructed above. The models with $40M_\odot$ and $100M_\odot$ are taken from the results of evolutionary calculations, while the latter model is Population III star. The model with $375M_\odot$ is taken from our systematic calculations. Since the collapse of the model with $100M_\odot$ had not been studied yet, we have computed its collapse under the equation of state by Shen et al (1998a, 1998b) at first. Comparing with our former investigations, we have found that the steep decline of the neutronization burst depends not only on the initial entropy but also on the initial density profile. Next, we have computed their collapse including quarks and pions, and found that the interval time from the bounce to the black hole (apparent horizon) formation becomes shorter. This is because they make the equation of state softer and the maximum mass becomes smaller. The temperature of collapsing matter decreases in the hadron-quark mixed phase owing to the assumption of the first-order transition. Moreover, quarks and pions affect the total energy of the emitted neutrinos because of the shorter duration. We have also shown that the neutrino luminosity becomes higher under the effect of pions because they raise the density of the proto-neutron star core. These features are same for the model with $40M_\odot$. As for the model with $375M_\odot$, the transition does not make a difference at all because it collapse to black hole directly. However, for the models with bounce, we stress that we may be able to discuss the equation of state of hot dense matter detecting the neutrinos from black hole progenitors in future.
Appendix A

Convergence Check

In this section, we compute models with higher resolutions so as to assess the convergence of our results. For normal models, we have used 127 radial mesh points, while 12 and 4 mesh points are used for energy- and angular distribution of neutrino, respectively. Models with higher resolutions have the same initial conditions as model E2. For model E2m, the number of radial mesh points is increased to 255. Model E2e uses 18 mesh points for the energy spectrum while model E2a has 6 mesh points for the angular distribution.

We show the results for the models with higher resolutions in Table A.1. The central density and the adiabatic index at bounce, which are key parameters in our analysis, are not very different for models E2, E2a and E2m. The central density at bounce of model E2e, which has 1.5 times finer energy mesh, is different by 14% from that of model E2. This is because neutrinos affect the entropy variations before the neutrino trapping. In fact, the central entropy at bounce of model E2a is $3.50 k_B$ while that of model E2e is $3.62 k_B$. However, qualitative features of their bounces are not changed. On the other hand, the interval times from the bounce to the apparent horizon formation

<table>
<thead>
<tr>
<th>model</th>
<th>$s_{\text{initial}}$ ($k_B$)</th>
<th>$M_{\text{iron}}$ ($M_\odot$)</th>
<th>$\rho_{\text{initial}}$ ($g \text{ cm}^{-3}$)</th>
<th>$T_{\text{initial}}$ (K)</th>
<th>$M_{\text{bounce}}$ ($M_\odot$)</th>
<th>$\rho_{\text{bounce}}$ ($g \text{ cm}^{-3}$)</th>
<th>$T_{\text{bounce}}$ (MeV)</th>
<th>$\gamma_{\text{bounce}}$</th>
<th>$t_{\text{recollapse}}$ (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E2</td>
<td>4.0</td>
<td>3.49</td>
<td>$1.40 \times 10^8$</td>
<td>7.75 $\times 10^9$</td>
<td>1.10</td>
<td>9.58 $\times 10^{14}$</td>
<td>26.9</td>
<td>1.89</td>
<td>62.0</td>
</tr>
<tr>
<td>E2m</td>
<td>4.0</td>
<td>3.49</td>
<td>$1.40 \times 10^8$</td>
<td>7.75 $\times 10^9$</td>
<td>1.04</td>
<td>9.64 $\times 10^{14}$</td>
<td>27.5</td>
<td>1.88</td>
<td>69.5</td>
</tr>
<tr>
<td>E2a</td>
<td>4.0</td>
<td>3.49</td>
<td>$1.40 \times 10^8$</td>
<td>7.75 $\times 10^9$</td>
<td>1.05</td>
<td>9.66 $\times 10^{14}$</td>
<td>27.2</td>
<td>1.88</td>
<td>65.7</td>
</tr>
<tr>
<td>E2e</td>
<td>4.0</td>
<td>3.49</td>
<td>$1.40 \times 10^8$</td>
<td>7.75 $\times 10^9$</td>
<td>1.10</td>
<td>8.25 $\times 10^{14}$</td>
<td>25.9</td>
<td>1.80</td>
<td>73.2</td>
</tr>
</tbody>
</table>

Notations are same with Table 4.3
Figure A.1: Luminosities of $\nu_e$ as a function of $t$ for models E2 (solid line), E2m (short-dashed line), E2e (long-dashed line) and E2a (dot-dashed line). The meaning of squares is the same as in Figure 4.5.

are different by $\lesssim 15\%$ for models E2, E2e, E2a and E2m. This is because the start point of the recollapse is roughly determined by the maximum mass of the neutron star as mentioned already. Since the mass accretion rate is of the order of $10M_\odot s^{-1}$ during this phase in our models, the difference of $0.1M_\odot$ in the maximum mass means the difference of $10\text{ ms}$ in the interval time, which is close to the discrepancies found here. Thus, the precise determination of the interval time is difficult in general. However, its dependence on the initial entropy is well established.

The results for the time evolutions of the neutrino luminosity are shown in Figure A.1. While the duration times of their neutrino emissions differ slightly among the models as mentioned already, the profiles of their neutronization bursts are not very different qualitatively. In fact, the luminosity declines a little after the peak and increases again for model E2. This feature is well kept in other models with higher resolutions.
Appendix B

Neutrino Oscillation

In order to estimate the event numbers of neutrinos from black hole progenitor, we should take into account the neutrino oscillation. We introduce the general formulation of the flavor conversion by Mikheyev-Smirnov-Wolfenstein (MSW) effect (Wolfenstein 1978; Mikheyev & Smirnov 1985). Now the existence of neutrino oscillation is convincing from the recent data of solar and atmospheric neutrinos (e.g., Fukuda et al. 1999; Fukuda et al. 2001). It is noted that neutrinos emitted from stellar collapse propagate through the stellar envelope and the earth, where neutrino flavor conversion occurs by the MSW effect. Flavor conversion of the supernova neutrino is now well studied (e.g., Dighe & Smirnov 2000; Takahashi et al. 2001; Fogli et al. 2002; Takahashi et al. 2003a) and there are studies also taking into account the shock propagation (Takahashi et al. 2003b), the neutrino self-interaction (Duan et al. 2006) and the resonant spin-flavor conversion (Ando & Sato 2003). Here, we evaluate the flavor conversion inside the stellar envelope using the results of evolutionary calculations for the stars with $40M_\odot$ (Woosley & Weaver 1995; Hashimoto 1995) and $50M_\odot$ (Umeda & Nomoto 2005; Tominaga et al. 2007) towards the oscillation of neutrinos from the “black hole progenitors”. Incidentally, these models are initial conditions of the numerical simulations in Sumiyoshi et al. (2007, 2008). We regard $40M_\odot$ model by Woosley & Weaver (1995) as a reference model later, which is same as model X in Chapter 6. We also show the earth effects are computed from realistic density profile of the earth (Dziewonski & Anderson 1981).
B.1 General Formulation of Flavor Conversion by MSW Effect

In this section, we review the neutrino oscillation of MSW effect in brief. Neutrino oscillation is caused by the discrepancy between the mass eigenstate and flavor eigenstate of neutrino. In general, the flavor eigenstate is related with the mass eigenstate as below,

\[
|\psi\rangle \equiv \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},
\]  

(B.1a)

\[
U = \begin{pmatrix}
  c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
  -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\
  s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}
\end{pmatrix},
\]  

(B.1b)

where \( s_{ij} = \sin \theta_{ij} \), \( c_{ij} = \cos \theta_{ij} \) and \( \theta_{ij} \) is the mixing angle for \( i, j = 1, 2, 3 \) \((i < j)\). While \( \sin^2 \theta_{12} = 0.32 \) and \( \sin^2 \theta_{23} = 0.5 \) are well measured from recent experiments, only the upper limit is given for \( \sin^2 \theta_{13} \leq 2.0 \times 10^{-2} \) (Gonzalez-Garcia & Maltoni 2007). Neutrino propagation is described by the Dirac equation for the mass eigenstate. Moreover, electron type neutrinos get the effective mass from the interaction with electrons when they propagate inside a matter. Thus, the time evolution equation of the neutrino wave functions can be written as,

\[
\frac{id}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U^{-1} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21}/2E & \Delta m^2_{31}/2E \\ 0 & 0 & \Delta m^2_{31}/2E \end{pmatrix} U \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F n_e(t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix},
\]  

(B.2)

where \( G_F, n_e(t) \) and \( E \) are the Fermi constant, the electron number density and the neutrino energy, respectively. In case of the anti-neutrino sector, the sign of \( n_e(t) \) changes. \( t \) is an Affine parameter along the neutrino world-line. \( \Delta m^2_{ji} \) are the mass squared differences and they are experimentally determined as \( \Delta m^2_{21} = 8 \times 10^{-5} \) eV and \( |\Delta m^2_{31}| = 3 \times 10^{-3} \) eV. Whether the sign of \( \Delta m^2_{31} \) is plus (normal mass hierarchy) or minus (inverted mass hierarchy) is unclear under the current status (Gonzalez-Garcia & Maltoni 2007). Solving equation (B.2) along the neutrino propagation, we can follow the neutrino flavor eigenstate \( |\psi\rangle \) and evaluate the survival probability of \( \nu_f \) \((f = e, \mu, \tau \text{ and their anti-particles})\) as \( p_{\nu_f} = |\langle \psi_{\nu_f} | \psi \rangle|^2 \).
A calculation of the survival probability can be divided into two parts, namely a probability that a neutrino generated as flavor $f$ reaches to the earth with $i$-th mass eigenstate, $P^\star(\nu_f \to \nu_i)$, and a probability that a neutrino entering the earth with $i$-th mass eigenstate is detected as $\nu_f$, $P^\oplus(\nu_i \to \nu_f)$. Using them, the survival probabilities are expressed as,

$$p_{\nu_e} = \sum_{i=1}^{3} P^\star(\nu_e \to \nu_i)P^\oplus(\nu_i \to \nu_e), \quad (B.3a)$$

$$p_{\bar{\nu}_e} = \sum_{i=1}^{3} P^\star(\bar{\nu}_e \to \bar{\nu}_i)P^\oplus(\bar{\nu}_i \to \bar{\nu}_e), \quad (B.3b)$$

$$p_{\nu_a} \approx 1 - \sum_{i=1}^{3} P^\star(\nu_\mu \to \nu_i)P^\oplus(\nu_i \to \nu_e),$$

$$\approx 1 - \sum_{i=1}^{3} P^\star(\nu_\tau \to \nu_i)P^\oplus(\nu_i \to \nu_e), \quad (B.3c)$$

$$p_{\bar{\nu}_a} \approx 1 - \sum_{i=1}^{3} P^\star(\bar{\nu}_\mu \to \bar{\nu}_i)P^\oplus(\bar{\nu}_i \to \bar{\nu}_e),$$

$$\approx 1 - \sum_{i=1}^{3} P^\star(\bar{\nu}_\tau \to \bar{\nu}_i)P^\oplus(\bar{\nu}_i \to \bar{\nu}_e), \quad (B.3d)$$

where $\nu_a$ represents the sum of $\nu_\mu$ and $\nu_\tau$, and $\bar{\nu}_a$ represents the sum of $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$. In other words, $p_{\nu_a}$ is a probability that the neutrino generated as $\nu_\mu$ is detected as $\nu_\mu$ or $\nu_\tau$ on the earth. It is noted that we can assume $\mu$-$\tau$ symmetry owing to $\sin^2 \theta_{13} \approx 0$ and $\sin^2 \theta_{23} = 0.5$.

**B.2 Flavor Conversion inside Stellar Envelope**

Neutrinos emitted from the core of black hole progenitor propagates through the stellar envelope, which has the density gradient. In other words, the electron number density, $n_e(t)$, varies along the neutrino worldline. It is noted that the conversions occur mainly in the resonance layers. The matter density at the resonance layer is given as,

$$\rho_{\text{res.}} \sim 6.6 \times 10^5 \left( \frac{\Delta m^2}{1\text{eV}^2} \right) \left( \frac{20\text{MeV}}{E} \right) \left( \frac{0.5}{Y_e} \right) \left( 1 - 2 \sin^2 \theta \right) \text{g cm}^{-3}, \quad (B.4)$$

where $Y_e$ is the electron fraction. In the stellar envelope, there are two resonances, namely H resonance and L resonance, where H resonance lies at
higher density ($\sim 2 \times 10^3 \text{ g cm}^{-3}$) than L resonance does lower ($\sim 20 \text{ g cm}^{-3}$). H resonance corresponds to the case of $\Delta m^2 = \Delta m^2_{31}$ and $\theta = \theta_{13}$ in equation (B.4), while L resonance does to $\Delta m^2 = \Delta m^2_{21}$ and $\theta = \theta_{12}$. Thus, in case of the normal mass hierarchy, both resonances reside for the neutrino sector and the anti-neutrino sector has no resonance. On the other hand, in case of the inverted mass hierarchy, H resonance shifts to the anti-neutrino sector. If the density profile is shallow at the resonance layer, the resonance is “adiabatic” and the conversion occurs completely. When the resonance layer has a steep density profile, the resonance is “non-adiabatic” and the conversion does not occur. The flip probability of the mass eigenstates, $P_F$, is analytically estimated (Bilenky & Petcov 1987) as

$$P_F = \frac{e^{\frac{\chi(1 - \sin^2 \theta)}{e^\chi - 1}} - 1}{e^\chi - 1},$$

(B.5a)

$$\chi \equiv -2\pi \left[ \frac{\Delta m^2}{2E} \left\{ \frac{1}{n_e(r)} \frac{dn_e(r)}{dr} \right\} \right]_{r=r_{\text{res}}},$$

(B.5b)

where $r$ and $r_{\text{res}}$ are the radial coordinate and the position of the resonance layer, respectively.

Using flip probabilities at H resonance ($P_H$) and L resonance ($P_L$), $P^*(\nu_f \rightarrow \nu_i)$ in equations (B.3) are calculated. Here we define matrices $P^*$ and $P^*_{\text{if}}$ whose elements, $P_{ij}^*$ and $P_{\text{if}}^*$, are $P^*(\nu_f \rightarrow \nu_i)$ and $P^*(\bar{\nu}_f \rightarrow \bar{\nu}_i)$, respectively, and they are calculated (Dighe & Smirnov 2000) as

$$P^* = \begin{pmatrix} P_L P_H & 1 - P_L & P_L(1 - P_H) \\ (1 - P_L) P_H & P_L & (1 - P_L)(1 - P_H) \\ 1 - P_H & 0 & P_H \end{pmatrix},$$

(B.6a)

$$\overline{P^*} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(B.6b)

in case of the normal mass hierarchy. On the other hand, in case of the inverted mass hierarchy, H resonance resides for the anti-neutrino sector and the survival probabilities are given as,

$$P^* = \begin{pmatrix} P_L & 1 - P_L & 0 \\ 1 - P_L & P_L & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(B.7a)

$$\overline{P^*} = \begin{pmatrix} P_H & 0 & 1 - P_H \\ 0 & 1 & 0 \\ 1 - P_H & 0 & P_H \end{pmatrix}. $$

(B.7b)
Figure B.1: Neutrino survival probabilities for the reference model with $E = 20$ MeV. Plots represent results of the numerical computations and lines do the analytic formula.

In the absence of the earth effects, $P^\oplus(\nu_i \rightarrow \nu_f)$ is displaced to $|U_{fi}|^2$, where $U_{fi}$ are elements of matrix $U$ given in equation (B.1b), and equation (B.3) are rewritten as,

\[ p_{\nu_e} \approx P_H \left[ (1 - \sin^2 \theta_{12}) P_L + \sin^2 \theta_{12} (1 - P_L) \right], \quad \text{(B.8a)} \]
\[ p_{\bar{\nu}_e} \approx 1 - \sin^2 \theta_{12}, \quad \text{(B.8b)} \]
\[ p_{\nu_\alpha} \approx 1 + P_H \left[ (1 - \sin^2 \theta_{12}) P_L + \sin^2 \theta_{12} (1 - P_L) \right], \quad \text{(B.8c)} \]
\[ p_{\bar{\nu}_\alpha} \approx \frac{2 - \sin^2 \theta_{12}}{2}, \quad \text{(B.8d)} \]
in case of the normal mass hierarchy and

\[ p_{\nu_e} \approx (1 - \sin^2 \theta_{12}) P_L + \sin^2 \theta_{12} (1 - P_L), \quad \text{(B.9a)} \]
\[ p_{\bar{\nu}_e} \approx P_H (1 - \sin^2 \theta_{12}), \quad \text{(B.9b)} \]
\[ p_{\nu_\alpha} \approx \frac{1 + (1 - \sin^2 \theta_{12}) P_L + \sin^2 \theta_{12} (1 - P_L)}{2}, \quad \text{(B.9c)} \]
\[ p_{\bar{\nu}_\alpha} \approx \frac{1 + P_H (1 - \sin^2 \theta_{12})}{2}, \quad \text{(B.9d)} \]
in case of the inverted mass hierarchy. In Figure B.1 we show the results for the survival probabilities as a function of $\sin^2 \theta_{13}$ for the reference model.
with \( E = 20 \) MeV. In this figure, the analytic formula, equations \((\text{B.8})\) and \((\text{B.9})\), are compared with the numerical solutions of equation \((\text{B.2})\) by Runge-Kutta method. We can recognize that the analytic estimations well coincide with numerical results. This trend holds true for other progenitor models and neutrino energies, \( E \), and we can safely use the analytic formula hereafter. In addition, L resonance is perfectly adiabatic (i.e., \( P_L \approx 0 \)) for all of the progenitor models adopted here and for an energy range of our interest \((E = 1-100 \) MeV\)). Thus, in case of the normal mass hierarchy \( p_{\bar{\nu}_e} \approx 0.68 \) and \( p_{\bar{\nu}_a} \approx 0.84 \) are robust while \( p_{\nu_e} \approx 0.32 \) and \( p_{\nu_a} \approx 0.66 \) are in the inverted mass hierarchy.

### B.3 Progenitor Dependence

In Sumiyoshi et al. (2008), the progenitor dependence of neutrino emission is investigated. We utilize the same stellar models of evolutionary calculations with Sumiyoshi et al. (2008) in order to examine the model dependence of the neutrino survival probabilities. \( 40M_\odot \) model by Woosley & Weaver (1995) (the reference model), \( 40M_\odot \) model by Hashimoto (1995) and \( 50M_\odot \)
Figure B.3: Energy dependence of the survival probabilities of $\nu_e$ for each progenitor model. Dotted, solid, long-dashed, short-dashed and dot-dashed lines represent the cases of $\sin^2 \theta_{13} = 10^{-2}, 10^{-4}, 10^{-5}, 10^{-6}$ and $10^{-8}$, respectively. Here the normal mass hierarchy is posited.

The mass-loss rate is thought to depend on the stellar metalicity, $Z$. Since TUN07 is a model with $Z = 0$, the effects of mass-loss are not included. Incidentally the same model with TUN07 but $Z = 0.02$ (solar metallicity) has also been computed (Umeda & Nomoto 2007). In this paper, we also adopt this model as TUN07Z in order to discuss its sensitivity. In addition, WW95 and H95 are solar metalicity models but without the mass loss.

As already seen in equation (B.4), the resonance layer depends on the
neutrino energy, $E$. In Figure B.3, the energy dependences of $p_{\nu_e}$ for each progenitor model are shown in case of the normal mass hierarchy. $p_{\nu_e}$ is insensitive to the progenitor model for $\sin^2 \theta_{13} \geq 10^{-2}$ or $\sin^2 \theta_{13} \leq 10^{-8}$ whereas it is sensitive for $10^{-8} \leq \sin^2 \theta_{13} \leq 10^{-2}$. In case of intermediate value of $\sin^2 \theta_{13}$, there are energies which give a peak of $p_{\nu_e}$ for some models. Roughly speaking, H resonance of neutrinos with this energy lies at the boundary of layers. For instance, model TUN07 has the boundary of convective region and nonconvective region near He shell burning layer. On the other hand, the envelope of model TUN07Z is already stripped and there are not such a boundary near the resonance layer. Thus, its result does not have a peak of $p_{\nu_e}$ even for the case of intermediate value of $\sin^2 \theta_{13}$. These trends hold true for $p_{\nu_{\mu}}$ and, in case of the inverted mass hierarchy, $p_{\bar{\nu}_{e}}$ and $p_{\bar{\nu}_{\mu}}$.

### B.4 Earth Effects: Nadir Angular Dependence

In this paper, we evaluate the earth effects, $P^\oplus(\nu_i \rightarrow \nu_f)$, solving equation (B.2) along the neutrino worldline numerically by Runge-Kutta method using the realistic density profile of the earth (Dziewonski & Anderson 1981). It is noted that the density variation along the neutrino worldline depends
the nadir angle of the black hole progenitor. In Figure B.4, we show some examples for the results of the earth effects. We find that the spectral shape is deformed to wave-like and varies with the nadir angle. If the value of $\sin \theta_{13}$ is larger ($\gtrsim 0.01$), this effect disappears for the neutrino sector in case of the normal mass hierarchy and for the anti-neutrino sector in case of the inverted mass hierarchy. However, the deformation of the spectral occurs for other parameter sets. These features are very similar with the case of the ordinary supernovae (Lunardini & Smirnov 2001; Takahashi et al. 2002) and ONeMg supernovae (Lunardini et al. 2007).
Bibliography


[86] Prigogine, I., & Defay, R. 1952, in Chemical Thermodynamics (London: Longmans Green and Co.)


